Teachers
Investigating
Adult
Numeracy

TIAN in Action: Teachers' Stories

TEACHERS INVESTIGATING ADULT NUMERACY

TIAN in Action: Teachers' Stories

Edited by Donna Curry
and Mary Jane Schmitt

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Foreword: Learners All of Us

TIAN is the Chinese word for "field" or "cultivated field". This book is dedicated to all teachers who work in the field of adult numeracy.

While many of us consider ourselves teachers, we are first and foremost learners, even when we think we are teaching.

We asked TIAN teachers to become learners while in the Institutes, and even when they went back to their own classrooms. They did so, and took risks to gain new strategies and knowledge in order to help their learners. They learned about ideas in math that are important for helping students succeed.

Every teacher participating in TIAN shared new learnings with us. All are worthy of sharing with you. We asked one or two teachers from each TIAN state to share an example from their own classroom, giving you, the reader, an opportunity to learn new strategies yourself. Here's a summary:

Falencia Bias (Literacy Council of Southwest Louisiana, Lake Charles, LA) focuses on several strategies for communicating in a math classroom.

Charlotte Brungardt (Barton Community College's Center for Adult Education in Great Bend, KS—now at Dominican Sisters of Peace) uses graphs as a communication form as she helps students make connections between graphs and the stories they tell.

Sarah Fearnow (Pima Community College Adult Education, Tucson, AZ) illustrates how students who are learning their multiplication facts can still engage in rich math activities involving geometry and algebra.

Lynn Foley (Project RIRAL (RI Regional Adult Learning), Woonsocket, RI) uses questioning techniques to prompt her students to think deeply about what they know and don't know.

Holly Lee (Great Oaks Career Development Campus, Cincinnati, OH) shares how she learns so much from her students by simply listening and posing effective questions.

Marty Lopinto (Great Oaks Career Development Campus, Cincinnati, OH) uses a variety of strategies to integrate number sense and algebra and to support sense making.

Abby Magee (Notre Dame Education Center, Boston, MA) demonstrates how critical communication is in a math class, especially when students come from different countries.

The most important thank-you has to go to adult learners across the country. We have committed to this project because of them. We believe in them, know that they are capable of understanding—and even enjoying—math if given the chance to think for themselves, to express their own ideas, to apply strategies that make sense to them, and to build on what they already know.

Enjoy reading!
Donna Curry and Mary Jane Schmitt
Editors and TIAN Co-directors

Introduction:
The TIAN Four Big Ideas

Teaching math in adult education is a challenge. Despite motivation to reach goals such as passing the GED or going on to further education and training, adult students often bring the baggage of past experiences that has left them math-avoidant, if not math-anxious. They may return to school with a fragile number sense, the result of interrupted education. Once they do enroll in adult education, the demands of their lives compete with the need to persist in their studies. Adult education teachers talk about unsteady attendance, wide ranges of math ability and knowledge in classes, and students' belief that math is about memorizing rules. But there is a lot of research that says there are ways for teachers and students to change this—to create opportunities for learning mathematics that build upon and extend learners' out of school experiences and that are meaningful, useable, challenging, and engaging. In this book we have the stories of teachers who are investigating practices to that end. They represent a variety of perspectives of some of the more than one hundred teachers who have participated in the creation of the TIAN Project.

What are TIAN practices?
The Teachers Investigating Adult Numeracy (TIAN) Project is a professional development program piloted and field-tested with Adult Basic Education teachers in six states across the country. TIAN builds on previous efforts in the fields of mathematics education, adult education, career and workforce education, and teacher education. In order to set a new direction for high quality, effective in-service opportunities for adult education mathematics and numeracy teachers, the TIAN team grounded their work in promising practices and evidence from across educational levels from sources such as the National Council of Teachers of Mathematics Principles and Standards for School Mathematics (NCTM, 2000), the National Research Council's Adding It Up: Helping Children Learn Mathematics (2001), the Adult Numeracy Network Teaching and Learning Principles (ANN, 2005), and the Equipped for the Future (EFF) research and development of the adult content standard *Use Math to Solve Problems and Communicate* (NIFL, 2000).

The TIAN professional development intervention explicitly, with its materials, Institutes, and supporting activities, encourages teachers to experiment with classroom lessons that promote:

- Communication
- Connections
- Integration of math content strands at all levels
- A multi-dimensional definition of mathematical proficiency

These four practices, or "big ideas", are the focus of this document. First, we expand upon each idea and describe its significance to adult education math instruction. Then, we share examples of how these practices play out in actual classroom situations. Teachers from each of the six states (Arizona, Kansas, Louisiana, Massachusetts, Ohio, and Rhode Island) involved in the TIAN Project demonstrate the interplay of these four "big ideas" from their own perspectives.

Why *Communication* in Adult Education Math Instruction?

The act of talking and writing about math supports learning on many levels. For adults, the usefulness of mathematical communication goes beyond learning in the classroom. It extends to using skills in the workplace as well as other aspects of daily life. Employers want and need good communicators. Such communication is important for adults in the workplace where the ability to work in teams, interact and communicate effectively is placed at a high premium (U.S. Department of Labor, 1992). Adult learners need opportunities to listen to the ideas of others, to hold back judging, and to be willing to learn from others. Communication in the math classroom serves as an opportunity to do that and serves as well as serves as a strategy for teaching and learning math.

Communication between students can help them clarify and expand understandings and learn from one another. Communication among students – both oral and written – can help teachers uncover student understandings. By asking purposeful questions and listening well (with sufficient wait time), teachers can create a learning environment that honors students' contributions and encourages active participation in the learning process.

> "Instructional programs from pre-kindergarten through grade 12 should enable all students to
>
> - Organize and consolidate their mathematical thinking through communication;
> - Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
> - Analyze and evaluate the mathematical thinking and strategies of others;
> - Use the language of mathematics to express mathematical ideas precisely."
>
> (NCTM, *Principles and Standards for School Mathematics,* 2000, p. 60)

TIAN focused on two key adult education sources regarding communication in math. The EMPower curriculum (Schmitt, Steinback, Curry, Donovan, & Merson, 2005), from which many of the activities in TIAN are drawn, advocates for communication among all learners in the adult education classroom. The EMPower website asserts:

The EMPower pedagogy is focused on sets of connected activities that require communication and discourse. EMPower asks students to

- *Work collaboratively with others on open-ended investigations;*
- *Share strategies orally and in writing; and*
- *Justify answers in multiple ways.*

These three bulleted points were incorporated into the pedagogy of the TIAN professional development and they became pedagogical goals for teachers as well as students.

The Equipped for the Future Standards, another primary guiding tool for TIAN, explicitly incorporates communication as one of the purposes for adults learning math. The EFF math standard reads:

Use Math to Solve Problems and Communicate

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models (NIFL, 2000).

The TIAN staff and advisors explicitly focused on "communication" for two reasons. As just pointed out, communication is highly visible in current math education, workplace, and math education literatures. However, a second compelling reason is that student-to-student communication is noticeably absent in adult education classroom practice (both reported and observed); instead, individualized instruction or lecture is the norm. The TIAN Project staff and advisors believed that by increasing communication in the math classroom, student achievement and success would increase as well. All the stories shared by TIAN teachers support this assumption. Stories such as those by Falencia Bias and Lynn Foley illustrate how communication plays out between student and teacher and among students. And, Abby Magee shares how communication can be used to teach vocabulary in a multi-cultural math class.

Why *Connections* in Adult Education Math Instruction?

Across the United States, ask almost any adult education teacher what her students struggle with the most and you will hear, "Fractions, decimals, and percents." Typical workbooks teach these as isolated topics, one chapter at a time. Therefore, it is no surprise that adult learners often have not made connections among those three. Nor do they often know how to work with fractions, decimals, and percents in problems from the type posed in the workbook or in real life. Students struggle to make meaningful connections.

Making connections among mathematical ideas

The term "connections" summarizes the ways in which students build understanding by making linkages, both inside and outside mathematics. Take something simple, like "¾". Within mathematics, this common fraction connects to a multitude of concepts (e.g., part of a whole; a ratio; a division problem; a point between 0 and 1; an operator that shrinks another amount) and equivalent representations (0.75; 0,75 (in many countries); 6/8; 45/60, 75%, etc.). A robust vision of “three quarters” enables reasoning within geometry, measurement, data analysis, and algebra, where each of the concepts and representations can come into play.

Apply mathematics in contexts outside of mathematics

And, the connections outside of mathematics to familiar and important contexts go far beyond the ever-popular pizza. A statement such as “about ¾ of entering community college students take remedial math” is not easy to ignore because it is more than half and pretty close to all the students.

> “Instructional programs from pre-kindergarten through grade 12 should enable all students to
>
> - recognize and use connections among mathematical ideas;
> - understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
> - recognize and apply mathematics in contexts outside of mathematics”
>
> *(Principles and Standards for School Mathematics, 2000, p. 64).*

The authors of *Adding It Up: Teaching Math to Children* offer an instructional warning that the TIAN team took seriously:

> Connections are most useful when they link related concepts and methods in appropriate ways. Mnemonic techniques learned by rote may provide connections among ideas that make it easier to perform mathematical operations, but they also may not lead to understanding. These are not the kinds of connections that best promote the acquisition of mathematical proficiency.
> (p. 119)

Connections to and in contexts outside of mathematics, are essential as well. Studies in the United Kingdom, New Zealand, and Australia have explored the instructional practice of embedding instruction in the context of the real-life issues and concerns of students. A large-scale experimental project in the United Kingdom with young unemployed people on Youth Training Schemes compared different methods of teaching numeracy and problem solving. The study showed that using a range of contexts for teaching was most effective in improving trainees’ abilities to generalize their numeracy skills to new problems and situations (Wolf, Silver, & Kelson 1990).

In terms of connection specifically to work-related contexts, the literature suggests that mathematical knowledge does not qualify a worker for work unless it is integrated with knowledge, skills, and properties relevant to the practices and organization of the workplace (Wedege, 2000a). For example, Masingila (1992) studied carpetlayers as they solved problems encountered during installations. These constraint-filled situations differed substantially from the textbook area problems that the students were required to solve. The straightforward school problems did not prepare novice carpetlayers for the realities of area, ratio, proportion, and measurement experienced on the job.

The Equipped for the Future (EFF) initiative guides instruction that is connected to adults' roles and purposes. The EFF content standards identify three roles within which adults use mathematics: as worker, family member, and citizen. In the EFF framework, instruction and assessment are embedded in meaningful contexts that support learners in enacting their adult roles. The mathematics standard, as well as the other 15 EFF standards, states that an adult's acquisition of a particular proficiency like mathematics is connected to one or more purposes. In the case of EFF math, that purpose is to "use math to solve problems and communicate" (Stein, 2000, p. 35).

Key features of TERC's EMPower curriculum activities provide "contexts that are engaging and useful for young people and adults" (EMPower website **http://adultnumeracy.terc.edu/EMPower_home.html**). Lessons always include contexts, and most often lead with context. For example, lessons on linear functions are motivated by needing to figure out the best phone plan or job offer.

Throughout the project, TIAN staff encouraged participants to experiment with the EMPower lessons and to connect more authentically to student interests and concerns, using EFF protocols (e.g., the EFF Teaching/ Learning Cycle), which always launched from a student issue. As you read the teacher stories, pay attention to how teachers tried to help students make connections among math concepts such as fractions, decimals, and percents (see Marty Lopinto's story), connections to real-life contexts (such as Holly Lee's description), and connections between a story and the graph that illustrates it (Charlotte's example).

Why *Integration of Math Content Strands* at All Levels in Adult Education Math Instruction?

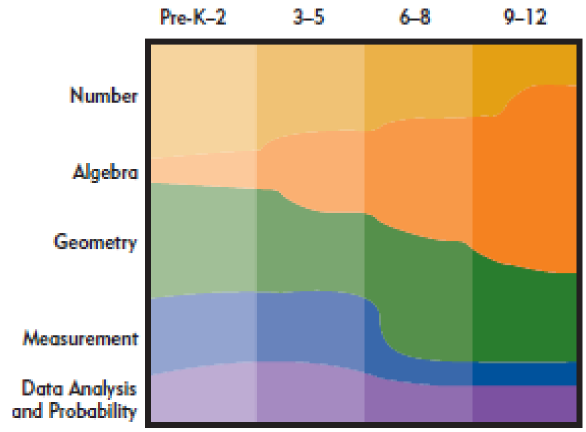
The adult basic education system faces the challenge of designing instruction for adults who do not remain in educational programs for extended amounts of time. Adults' needs for 'just-in-time' learning often do not align with the type of mathematics instruction found in most adult education programs. This traditional instruction is based on a linear sequencing of mathematics learning: computation procedures in sequence (addition, subtraction, multiplication, division, fractions, decimals, percents), then algebra, then geometry, and then data and statistics. Even if the order changes a bit from workbook to workbook, the topics are still isolated, sequenced chapters.

Research in K-12 education has shown that mathematics learning benefits from the simultaneous development of algebraic reasoning, measurement and geometry, and data and statistics throughout the course of instruction.

Applying math concepts and procedures requires an integration of skills from different strands. "Number, for example, pervades all areas of mathematics. Some topics in data analysis could be characterized as part of measurement. Patterns and functions appear throughout geometry." (NCTM, 2000, p. 31)

The National Council of Teachers of Mathematics emphasizes that the

Content Standards of Number, Algebra, Geometry, Measurement, and Data Analysis and Probability apply across all grade levels, even though they should receive different emphases across the grade bands.



The Content Standards should receive different emphases across the grade bands.

Referring to school-aged children, the authors of *Adding it Up* also argue for developing math proficiency "beyond number" at all levels, and cite much math education literature to support that suggestion. The school mathematics curriculum, they say, needs to be experienced by the learner as a unified whole.

These ideas make as much, if not more, sense for adult learners as they do for grade school students. Adults come to numeracy classes with varied schooling backgrounds: some have gone to school in other countries, some have repeatedly "reviewed" math with many teachers, and others have never been exposed to formal algebra or geometry, but have informal everyday mathematical experience. Acknowledging and drawing upon these experiences can motivate further study. Moreover, coming at math from a different angle can be more engaging than doing the same things in the same order that learners have been through before.

The TIAN project encouraged teachers to challenge the conventional sequence found in adult education materials. This change from the traditional sequence was guided and supported by the EMPower materials **(http://adultnumeracy.terc.edu**) that teachers used during TIAN and the EFF performance continuum (**http://eff.cls.utk.edu/PDF/EFFMathPC.pdf**), which provides guidance for developing algebra, geometry, number, and data lessons throughout the various adult education defined levels (NRS Levels). The expectations, of course, are not the same at each level. Development of each content area moves from more familiar, concrete contexts to more the formal and abstract. For example, the benchmark fraction ½ is taught at a very early level, using concrete manipulatives. Then, students develop fluency with other benchmark fractions. It is only after they have gained fluency with more common fractions that they begin to explore operations with messier fractions. To read more about benchmark fractions, read Marty Lopinto's story. Also, pay close attention to Sarah Fearnow's story as she shares the power of teaching geometry and algebra to students who are working on their multiplication and division facts.

Why a *Multi-dimensional Definition of Mathematical Proficiency* in Adult Education Math Instruction?

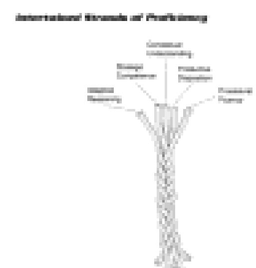
There is a sizable body of literature, in mathematics education, cognitive science, and adult workforce education supporting the notion that knowing how to perform mathematical procedures contributes to, but does not complete, a picture of mathematical competence.

In K-8 mathematics education, this notion has been expounded upon in the National Research Council's book, *Adding It Up*, which synthesized a large body of the research.

TIAN followed the lead of the Adult Numeracy Network (ANN) in their adoption of the National Research Council's definition. According to the ANN Teaching and Learning Principles (2005):

> "A high quality mathematics curriculum for adult learners should weave together *all* the elements of mathematical proficiency – not only procedural fluency, but also conceptual understanding, ongoing sense-making, problem solving, and a positive attitude about learning mathematics".

> "Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, we have chosen *mathematical proficiency* to capture what we believe is necessary for anyone to learn mathematics successfully. Mathematical proficiency, as we see it, has five components, or *strands:*
>
> *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
> *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
> *strategic competence*—ability to formulate, represent, and solve mathematical problems
> *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
> *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy"
>
> (National Research Council, *Adding It Up: Helping Children Learn Mathematics*, 2001, p, 115)



Mathematical proficiency strands

The TIAN Project team was keenly aware that the usual emphasis in adult education math classes had been on procedural knowledge. Evidence found in the instructional materials and our observations of teachers' classrooms bore this out. Too little time seemed to have been spent on developing conceptual understanding, solving non-routine problems, considering a variety of strategies, or creating opportunities to "think like mathematicians". Adult education teachers and students needed models for shoring up the other four strands in the mathematical proficiency rope. This was our belief, despite the counterpoint that adults have limited time to spend in class, and are in a rush to "get the GED". Teachers often used this as a reason for making the choice to short circuit a deeper development of a particular math topic. The TIAN Project challenged this choice and held the belief that this perspective was shortsighted. Adults get a GED for a reason, such as going on to career or further education. Learning with understanding will serve the immediate purpose of passing the test as well as the longer-term goal of meeting the math demands of a career choice.

The EMPower materials were developed to encourage adults to learn more than just procedures. Books such as *Many Points Make a Point: Data and Graphs* and *Seeking Patterns, Building Rules: Algebraic Thinking* encourage students to explore topics in-depth, helping them develop a deeper understanding of key data and algebra concepts. At the same time, students explore a variety of strategies as they develop critical thinking. (Note Charlotte Brungardt's description of how she used an EMPower lesson to informally build students' understanding of line graphs.)

Equipped for the Future's standards and performance continuum support this robust definition of mathematical proficiency by starting with contextualized problems and attention to the purpose for which math is used. Using the EFF Standard *Use Math to Solve Problems and Communicate*, students apply different strategies and procedures, depending on the situation and its purpose.

In addition to the K-12 math education and the adult education literature, the workforce education literature confirms that mathematical literacy is more than arithmetic. There appears to be broad agreement among researchers and commentators that success in the workplace requires mathematical knowledge and strategic competence more wide-ranging than what is traditionally taught in school math classes. Effective functioning on the job involves not only a diverse set of mathematical and arithmetical skills, but also broader knowledge and skills related, for example, to the ability to allocate resources, handle scheduling, understand the role of quantitative information in the operation of systems, and use technological tools to quantify or display quantitative information (Forman & Steen, 1999; Mayer, 1992; Packer, 1997; U.S. Department of Labor, 1992).

As you read the teacher stories, notice how they push to find out what the present understanding of their students is, especially as evidenced by Lynn Foley's dialogue with students about their thinking and Sarah Fearnow's story of students learning more than just procedures as they created geometric models to understand formulae.

Teacher Stories

While these four big ideas, or practices, may seem overly theoretical, they play out nicely in the classrooms of teachers who have embraced TIAN. The stories that follow are from seven teachers representing the six original TIAN pilot and field test states. They have graciously shared their classroom experiences in hopes that other teachers can understand how key the four principles are to successful teaching. In the sidebars of each of the stories, we have tried to point out where a key big idea is being illustrated.

These teachers also are expert in managing the challenges of adult education environments such as open enrollment and multi-level classes. Because these are challenges that all adult education teachers face, we also use the sidebars to highlight effective classroom management practices.

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Communication is Vital (and worth the time)

by Falencia Bias

Since 2006, Falencia Bias has been the program director for the Literacy Council of Southwest Louisiana, a non-profit literacy organization. She oversees the Adult Literacy program which offers classes in Workplace Computer skills, GED, ESL, and Prison Literacy, as well as one-to-one tutoring. She started with the Council in 2002 as an AmeriCorps Vista volunteer and in 2003, became the family literacy coordinator and the computer instructor. Her story below focuses on her role as morning GED instructor, which she has been since 2004. Her class includes Beginning ABE to Low Adult Secondary students who range in age from 16 to 80+ years. Last year, the program moved to "managed-open enrollment" where students are only tested on certain days during the week and then students start class. Falencia shared, "Two years ago we were happy to have 5-8 students on any given day with 20 being active at any given time. Last year, my GED class had 18-20 students a day with an average of 35-40 being active at any given time. This has led us to create two separate classes in the morning."

In my five years of teaching, I have tried a variety of instructional methods in an effort to figure out what is the most effective way of facilitating math classes. Like most adult education teachers, I get adult learners of all ages and many have a deep rooted fear of math. I am not a classically trained or certified teacher, but I have a bachelor's degree in Biology, a solid math background, and a strong desire to see my students succeed in life. I have learned that not only do I, as an adult education math instructor, need to be adaptable and flexible, but I must also be able to communicate math effectively to my students and provide time in class for students to communicate their newly obtained knowledge back to me and to their peers. I believe effective communication in math instruction helps to reduce, if not eliminate, fears adult learners have where math is concerned. They probably won't grow to love math, but they will develop a better understanding of math concepts and feel more comfortable when working with math problems.

I feel the thing that makes the difference in my adult education classroom is that we make communication a key part of class everyday. It is my job to teach students how to think for themselves and how to get to the answer themselves rather than me giving them the answer.

Teachers provide opportunities for students to write in math.

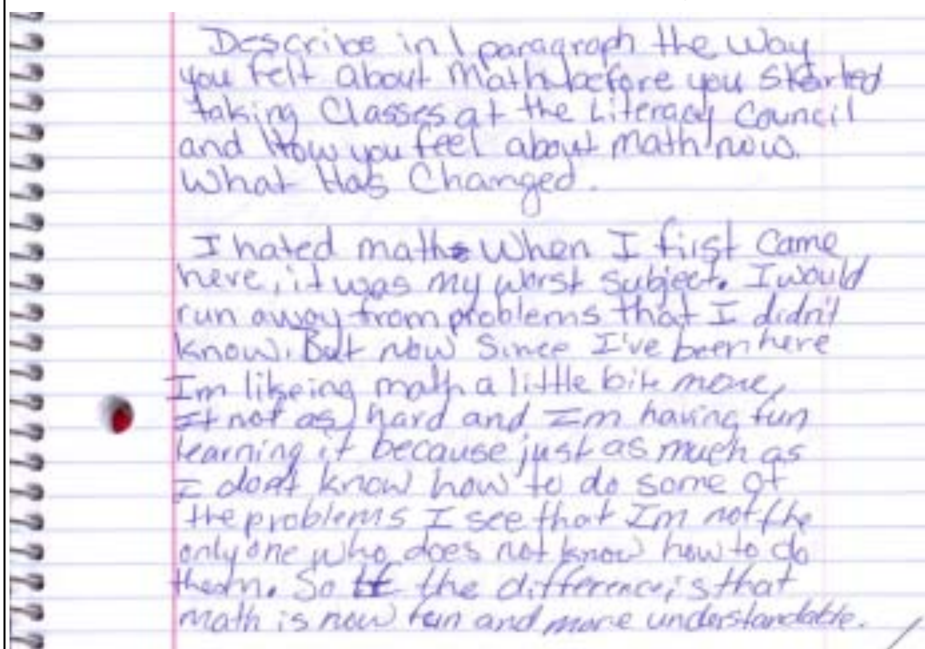
Like many math instructors, I found it difficult at first to understand how providing opportunities for students to write in math could be beneficial. In

Journaling in math is an effective strategy to encourage students to explain their thinking in math.

order to test the theory, I decided to do to writing activities with my students. The first was to have them write a math journal entry and the second was to take a problem from our daily review and write me a step by step process on how they would explain to their classmates how to solve the problem and then evaluate each others' explanations.

Math Journal

For the math journal, I asked my students to tell me how they felt about math before they came to the Literacy Council, how they now feel about math, and what made their feelings about math change. I was amazed that many of the student responses were similar. Many of the students admitted to hating or disliking math and confessed that math was a struggle for them to understand before coming to the Council. Many, like the student below, confessed that they still did not like math, but that they now understand math better and are a little more confident when it comes to math and solving problems.



Describe in 1 paragraph the way you felt about Math before you started taking Classes at the Literacy Council and how you feel about math now. What Has Changed.

I hated math when I fist came here, it was my worst subject. I would run away from problems that I didn't know. But now since I've been here Im likeing math a little bit more, it not as hard and Im having fun learning it because just as much as I don't know how to do some of the problems I see that Im not the only one who does not know how to do them. So the difference is that math is now fun and more understandable.

Another student's journal entry really stood out to me. She said the difference for her was that I just did not give her the answer when she had a question, but I asked her questions and then guided her to understand step-by-step how to work through the problems. I realized that what she was saying was that I just did not give an answer to her, but I also communicated the mathematical process with step-by-step instructions and questioned her as to what step would be next when we were making a problem in a way that helped her grasp math and understand problems she had not been able to solve in her 19 years. This student had been attending classes at the Literacy Council since August 2009 and entered as a Beginning ABE student. She had limited mathematical skills when she started and could only handle basic addition, subtraction, multiplication, and division of whole numbers. Her favorite phrase when asked a math question was, "I don't know." Today, she can handle fractions, decimals, signed numbers, basic algebra, and basic geometry. She is currently a High Intermediate ABE math student and I delight in hearing her tell new students, “Oh, that’s easy,” when I ask math questions she understands.

Step-by-Step Processes and Student Peer Evaluations

In another effort to test the benefits of writing in math, I had students select one problem from our daily review that they had solved correctly and write step-by-step how they would go about explaining the process to their fellow classmates. The objective was to see if my students could effectively communicate through writing how they had solved the problem, and to see if they could recognize a reasonable explanation and provide feedback to their classmates on how to improve an explanation. I expected that 90% of the students would read the explanations and have no opinion and think everything was great and that 10% would take the time to really evaluate the explanation and provide thoughtful input.

Students communicate their mathematical processes.

I collected their explanations once completed and made copies of them to distribute to the class. I asked the students to evaluate each explanation by answering the following questions:

1) Did you understand the explanation? Why or why not?
2) What advice would you give your classmate on how to make the explanation better?
3) How would you have explained the problem differently?

Students are encouraged to evaluate whether or not a particular process is reasonable.

I received my expected outcomes. 90% of the students agreed with their peer explanations and 10% were able to detect problems with the explanations.

I decided to take the lesson one step further the next day by picking one explanation and having the students solve the problem the way the problem was explained. I passed out one student’s explanation of an unlike denominator addition fraction problem. I explained to the students that they could only use the steps and information provided in the explanation to solve the problem. They could not use their knowledge of the math to solve.

It took 15 seconds before the complaining started. Students started saying, “This doesn’t make sense!” Students started listing the problems with the original explanation and verbally explaining what the person explained incorrectly. They were able to tell me what the person should have said. I had marked out the names on the paper so I explained that one of them had written the explanation and that just about everyone had agreed the day before that it was a reasonable and clearly written explanation.

This was a very useful experiment. I learned a great deal about what my students knew, and I was able to identify small areas of confusion regarding fractions with unlike denominators that I had missed during previous class sessions.

10-Question Daily Review

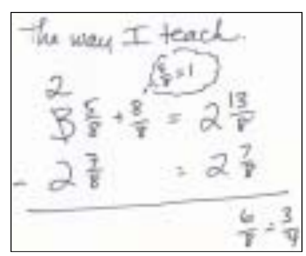
A typical class starts with a 10-question daily review that covers fractions to algebra. Students are given 15 minutes to solve the problems at their seats. They can work independently or in groups. Then I give everyone the opportunity to go to the board and solve problems. They can pick the problem or problems that they want to solve. The only requirement is that they write the steps on the board and explain to the class how they got their answers. If a student gets stuck on a step, I turn to the class and ask them to help their classmate out by asking questions of the class and the student at the board until the student is again comfortable taking over the explanation. At the end, I am always left with two or three questions that no one feels comfortable solving on the board so it becomes my time to go to the board. I don't go up and just solve the problem because I have learned that it doesn't benefit the students. The first thing I do is go to the board and ask the class in general what answers they came up with for the problem. I allow the students to compare answers without me providing any input. (This is usually easy for me to do since I generally just pull 10 problems out of the air that I have not solved previously, so I do not know the answer.)

Students compare their answers and share ways to solve problems. It's worth the time.

Since I don't proceed with a step until the students guide me, this method is long and does take time, but it helps the students. I learned that if I can get students to communicate with me as we move from step to step, they will eventually become confident enough in their skills to communicate how to solve problems with their classmates. When they can communicate the solving of problems or even just a few steps in the problem, that is more powerful to me then any test score. This shows me that they understand what is going on and the processes behind the math.

By the way, while I talk a lot about procedures and process, I always express to my students that in math there is usually more than one way to go about solving a problem. I always tell my students that we all view math in a different way and sometimes they may see ways of solving a problem that I don't see or that is not the "norm" and that it is okay. If a student expresses in class that he or she did not do the problem the way it was shown on the board, I ask that student to explain to me how he or she did the problem. If I'm shown a method that simplifies the problem or provides an alternative means of answering the problem that will always lead to the correct answer, I encourage that student to explain to the class what he or she did and give the other students the opportunity to choose which method makes most sense for them.

Teachers welcome students' alternative methods of finding the answer.



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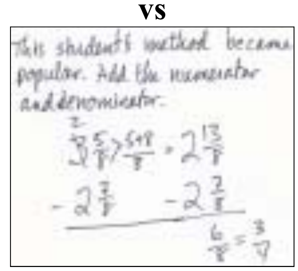


For instance, a few years ago we were working on subtracting fractions and borrowing. The class was struggling to understand what happened when they borrowed from the whole number and how the fraction then changed. One student expressed that she had learned a different method that was easier for them to understand. I gave her the marker and she demonstrated her strategy on the board. She had been taught to add the numerator and the denominator to get a new numerator and then put it over the old denominator to get her new fraction. The class loved this. Everyone who had been struggling with the traditional explanation decided to use this method. Today, I will use this method along with the traditional method when working with subtracting fractions and borrowing.

I generally have students explain the method to me first so that I can see if it works. When I have ESL students in class, I will allow them to share answers with the class. I also inform my ESL students that if they can solve the problems using their methods, they should not try to change and conform to our way of doing things.

Brain Teasers

Brain Teasers are a great way to open a class and foster the lines of communication. Students get the opportunity to do mathematical calculations, pull from prior knowledge, and – most importantly – think critically, problem solve, organize, and talk to one another. I think brain teasers are best done in a group session with the instructor facilitating and asking questions, if necessary, to force students to look deeper at the problem.

Brain teasers get students talking.

Recently, we did this brain teaser:

> Mrs. Ortiz made a batch of cookies for Carlos, Maria, Tina, and Joe. The children shared the cookies equally and finished them all right away. Then Mrs. Ortiz made another batch of cookies, twice as big as the first. When she took the cookies off the cookie sheet, 6 of them crumbled, so she didn't serve them to the children. She gave the children the rest of the cookies. Just then, Mr. Ortiz came home and ate 2 cookies from the children's tray. Each of the children ate 3 more cookies along with a glass of milk. They were stuffed, so they decided to leave the last 4 cookies on the tray.
>
> **How many cookies were in the first batch?**
> **How many cookies did each of the children eat?**

(Houghton Mifflin, http://www.eduplace.com/kids/mhm/brain/gr4/ch03/bt_04_03_q.html)

Everyone in class got the first part of the question correct by using prior knowledge. Prior knowledge led them to assume that cookies are typically baked in batches of 12. My question for them was, "How do you know she did not make really big cookies or really small cookies? How can you prove that the first batch had 12 cookies?" They could not answer this question immediately, nor could they answer the second part until the information was visually displayed on the board and everyone had had a chance to discuss the information in groups.

The brain teaser took 15 minutes to complete, but it provided time at the beginning of class get my students talking, it allowed them to relax and drop their defenses, and it served as a way to make old and new students comfortable asking me questions and offering their own input.

Instructor Vulnerability and Peer Answer Comparison

I understand that instructor vulnerability is a strange word to use for mathematical instruction, but it works. I describe instructor vulnerability as having a lesson to which I do not have the answers readily available. It makes me vulnerable to making small mistakes when solving problems. I've learned that if I'm not always perfect in class, it shows my students that anyone can a mistake and that it's OK. I encourage my students to disagree with my answers if they can justify why they are correct. If students call into question an answer on the board, we will work the problem together as a group. This allows for other students to agree or disagree with the student and then explain why. It also allows me to create a class environment where the students are required to compare answers aloud and then discuss with their classmate why one answer seems more reasonable than another.

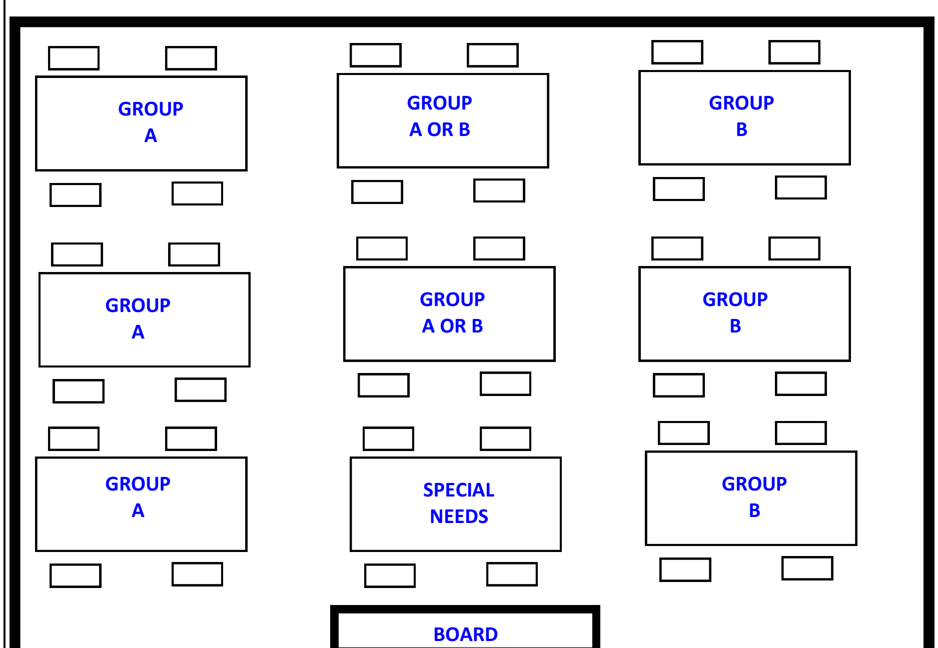
When I make a mistake at the board, it gives my more reluctant students the impetus they need to go to the board and try to solve a problem or call out an answer. They understand that it's OK to attempt solving a problem and not always get the right answer. The important thing is whether the student can look back on the problem and determine where he went wrong.

Students Communicating with Peers

In the last year, my GED class has ranged from 20-30 students, all at different NRS levels. The best way for me to meet the needs of my students was to divide them into two groups determined by NRS levels, and have each group sit together. One group was Beginning Literacy ABE to Low Intermediate ABE and the other High Intermediate to High Adult Secondary.

Students typically sit 2 or 4 to a table and are able to work together to complete their work, similar to the classroom arrangement below. The middle row is used for overflow from the two groups with the front table for special needs students who prefer to be closer to the board. Students typically sit 3 or 4 to a table when we are at maximum capacity.

Even seating arrangements can promote students learning from one another.

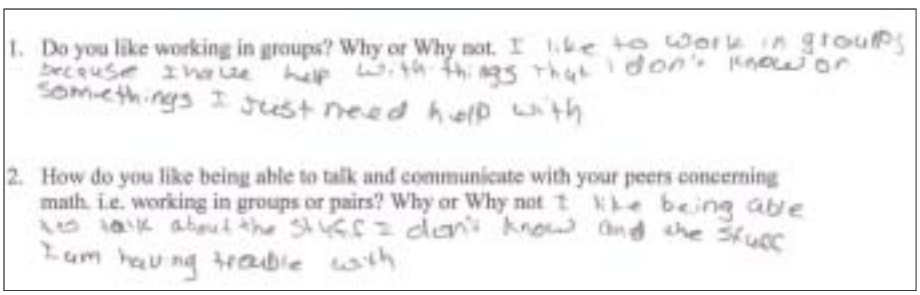


This situation works wonderfully with the higher group. I am able to introduce a mathematical topic to them and then they all work together to complete the work.

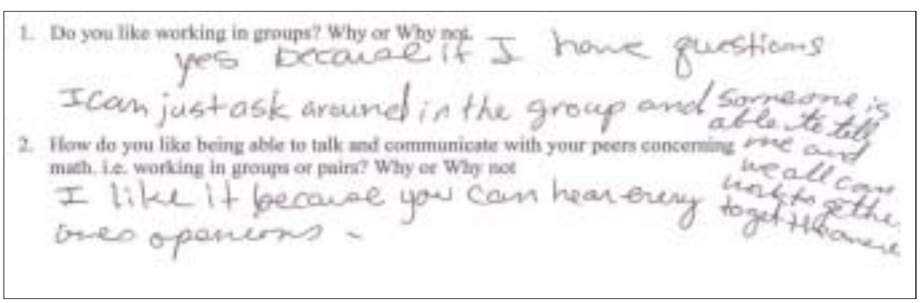
As I started to work with the lower group, I could hear the higher-level group asking each other questions and those that understood were taking the time out to share their knowledge. I noticed that as they continued to work, share, and communicate their knowledge that they became more confident. I would see students who never talked before suddenly open up in the small groups and then get praise from their peers on their knowledge. This even gave the younger students the chance to step up and assist their older classmates on mathematical skills that were fresher in the younger students' minds. This peer-to-peer communication built relationships with my students and I had one of the highest retention rates we have seen since we began offering classes four years ago.

I also saw positive effects with the lower-level group. I spent a great deal of time working with that group. Because I typically had ten to twelve in the group, I encouraged them to work in pairs or groups of three. If one student grasped a concept before the others in the group, I would leave that person in charge of helping the others complete their work by explaining the steps and mathematical processes. I believe this all helped to build self-esteem and reduce fear about math.

In fact, I asked the students whether they liked working in groups and communicating with their peers. Here is a sampling of responses.


1. Do you like working in groups? Why or Why not. I like to work in groups because I have help with things that I don't know or somethings I just need help with

2. How do you like being able to talk and communicate with your peers concerning math. i.e. working in groups or pairs? Why or Why not I like being able to talk about the stuff I don't know and the stuff I am having trouble with


1. Do you like working in groups? Why or Why not. yes because if I have questions I can just ask around in the group and someone is able to tell me and we all can work together the answer

2. How do you like being able to talk and communicate with your peers concerning math. i.e. working in groups or pairs? Why or Why not I like it because you can hear every ones opanions ~

Activity-Based Active Learning: Flooring

One way to encourage communication among peers is to engage them in project-based activities. For example, very recently, in an effort to present area, measurement, and percents in a different way to my students, I created a lesson that required them to purchase flooring for their own home. We reviewed area, measuring, and percents during our 10-question daily review.

Activity-based learning encourages student interaction; it also makes math real.

I then had the students answer questions individually about what they knew about those topics (see questionnaire below).

Pre- and post-questions allow students to quickly evaluate for themselves what they have learned through an activity. The pre-questions provide guidance about what needs to be taught in order for students to successfully perform the activity.

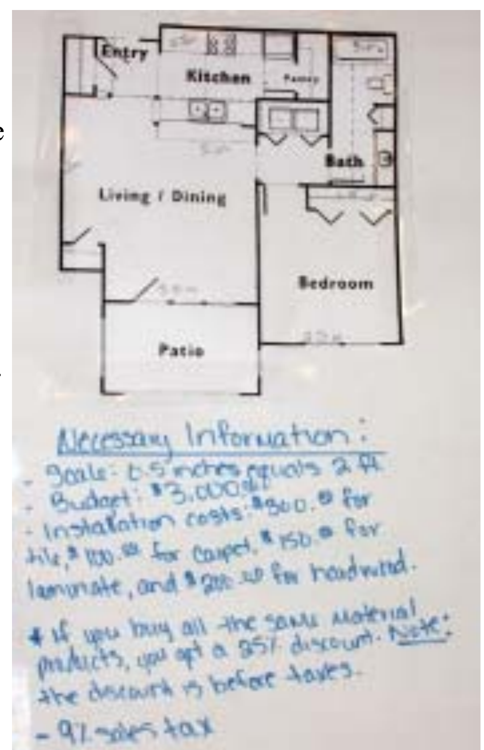
Using Area Calculations To Purchase Flooring

Pre-questions

1. How do you find a squared measurement such as Square Feet?
2. What is another way to write square feet?
3. Do you know how to use an inch ruler for measurement?
4. Do you know how to find the area of a regular figure?
5. Do you know how to find the area of an irregular figure?
6. What do you know about purchasing flooring for a house?

We then divided into groups and worked together on one problem involving area. Some students had problems using an inch ruler so their partners shared their knowledge. Except for one student, everyone struggled using a scale to convert their inches into feet. I demonstrated on the board, we talked about it, and then one student explained how he understood the conversion process. Once 50% of the class seemed to grasp the concept, they agreed as a whole to move forward with the project. I wanted to continue working on conversions until everyone understood, but I went with the flow thinking it would get the lines of communication open. I gave them the task to figure out the flooring using the floor plan on the right. They could choose between different flooring types, but had a limited budget.

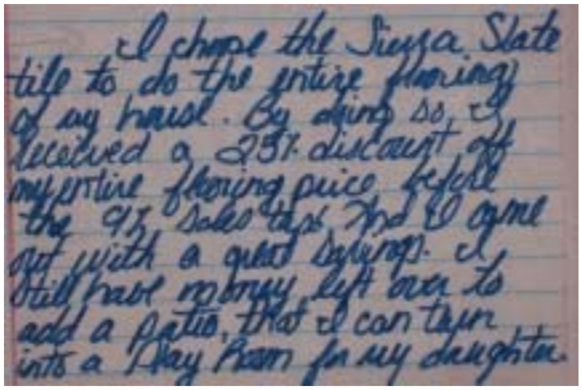
Students who feel some sense of ownership of the class are able to articulate when they are ready to move on.



The lesson was a success! Four different groups of students worked together, pulling together their knowledge to measure their floor plans and then convert their measurements to inches. One student who was really vocal at the beginning of the lesson about not understanding was so excited when she was finally able to understand the mathematical process once I drew a diagram to illustrate how the conversion worked. Once she grasped the concept, she went around to the other groups, helping them to convert their numbers correctly.

Students take care of the instructing.

The fun part came when the students had to then select tiling following a budget and guidelines. This meant they had to communicate with each other effectively so that they could make flooring choices that took into account budget, discounts, and even sales tax. They communicated orally and in writing (as shown by the student on the right who was thinking of her own personal experience as she worked through the activity).



We arranged everything on poster board for display in the classroom. (One group's work is shown on the right.) It was mad chaos, but a lot of learning and discussion was going on! The students were communicating with each other not only within their own small group, but also with other groups, providing assistance where needed. There were times where I was just able to stand back and hand them the tape and other materials they needed to prepare their poster. They were taking care of all the instructing themselves.

The teacher stands back and lets the students do the work. The students have a sense of responsibility for the success of the activity.

We concluded the day by answering a series of post-activity questions, similar to the ones I had asked before they began. I was pleased, as were the students, to see that they could answer many more questions at the end of the activity than at the beginning. This served as a nice informal assessment for the students and me. Everyone in class talked about how they enjoyed the lesson and how they had learned something.

These students worked for two hours straight completing their activity because they were engaged in the process.

The enthusiasm carried over into the next day. When I arrived at class, one group who had not finished the activity was hard at work already without being prompted to do so. Another student who had missed the activity was eager to begin once she saw the posters on the wall.

The success of the lesson was evident, because I did not have to explain the project to the student who missed the lesson. The rest of the class taught her the lesson! I did not even mind that they were doing math on a language and reading day.

Conclusion
I think communication in math class is vital to an adult learner's success. I believe instructors should provide ample time to allow for peer to peer communication and for students to communicate their newly obtained knowledge to their instructor. Anyone can give a learner a correct answer, but a true teacher can communicate math in a way that provides for the learner a deeper understanding of the mathematical process and allows for time for students to share that understanding with their classmates.

I believe there are multiple benefits for explicitly focusing on communication in the math classroom. They include improved self-esteem, a comfortable learning environment, learners who "get" that it is the understanding of the process and not the answer that is important, and learners who take ownership of their learning and lose their fear of math.

My students agree. When asked how they feel about math class now, a couple of students recently shared these thoughts.

I love how we help oneanother with problums in math.

I feel like Im learning more.

Sketching the Flow of a Story
by Charlotte Brungardt

Sister Charlotte Brungardt is a member of the Dominican Sisters of Peace and was a math instructor at the Barton Community College's Center for Adult Education in Great Bend, Kansas. She was a participant in Kansas TIAN I in 2006. She assisted with the facilitation of Kansas TIAN II (2007) and "Patterns, Functions, and Algebra", an Annenberg Course (2008), for other Kansas adult educators of math. A majority of the learners at the Center for Adult Education expressed a fear of studying mathematics at their time of registration. Sister Charlotte would strive to reduce the level of math anxiety in her students, finding that the students' experiences of success in the classroom would build more confidence in approaching further mathematics study. Lessons and activities offered in TIAN supported her efforts.

It was a wintry day. Three students traveling from a distance were absent. Two others called that day with family medical problems. This was the first week of a new class session. Of the six learners present that day, two were returning students, and four were in their first week. I was tempted to postpone the activity I had planned, but felt that this lesson could stand on its own and it would give me the opportunity to see more closely the four new students in action.

Adult ed teachers creatively manage attendance patterns.

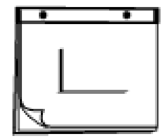
On the first day we had done a variety of introductory activities to explore learning preferences. The majority of the new students had shown that they were multi-modal with a slight preference to read/write, but also were kinesthetic. So with the entire group I knew I would want to be multi-modal in presentation and in activities. I found that the EMPower lessons lent themselves to that very easily – times for visual display and reading, times of oral input and discussion, and time for a student's own creating.

I chose EMPower's *Many Points Make a Point: Data and Graphs*: Lesson 5 - Sketch This. I selected this lesson because it seemed it could be fun and non-threatening for the new class. My challenge was to keep it light-hearted but instructive and engaging.

Graphs make frequent appearances on the GED in Mathematics, Science, and Social Studies.

All of these students were studying to prepare for the GED tests. Besides the math test that does include line graphs, the science and social studies tests also contain many kinds of graphs. I hope that students' confidence and performance will increase with more practice working with any kind of graph. Many of my students are looking at careers in nursing or business. These areas rely heavily on organizing data and relating information. Being able to write a good narrative of an event over the passage of time is a necessary skill. Making a simple sketch of the event that follows the flow can assist in this writing. Building a good descriptive vocabulary facilitates accurate communication.

The objectives of this lesson also tie to our state math standards (CASAS). These math standards look for students to extract information from line graphs, to make generalizations about data, to create graphs, and to identify trends.



From Stories to Graphs

Giselle's Baby

Giselle's baby's temperature rose sharply between 6:00 p.m. and midnight. Giselle gave her baby acetaminophen, and the baby's temperature dropped rapidly and stayed close to normal for most of the next day. At 6:00 p.m., however, it began to rise again, and Giselle was worried.

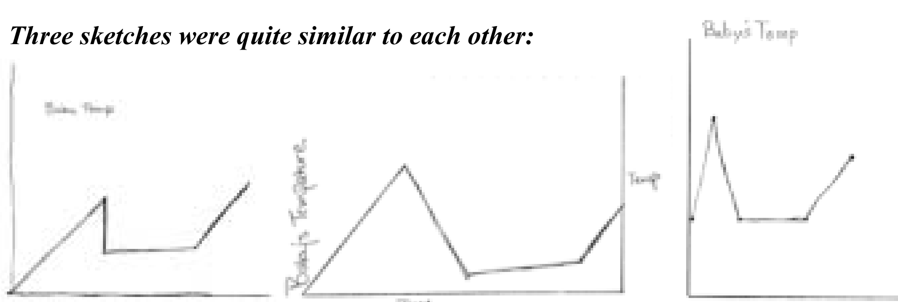
I explained to learners that this class period would be devoted to line graphs, which they could expect to meet on the GED. I asked them to take a piece of paper and using a ruler, draw a large "L" on that paper. I explained that this would be the start of a graph. I demonstrated on the board. Next, I asked them to listen to a short story and imagine the flow of the happenings. I read the story of "Giselle's Baby". I read it again and we then listed the significant happenings on the board. I asked them to draw a sketch within that "L" to illustrate the story's events.

They proceeded to do that and then tried to insert gridlines, scales, etc. on that paper. I asked them to stop and told them we would get to the specifics of the graph but for now just to draw the general flow of the story. Each started again on another sheet of paper to illustrate the narration.

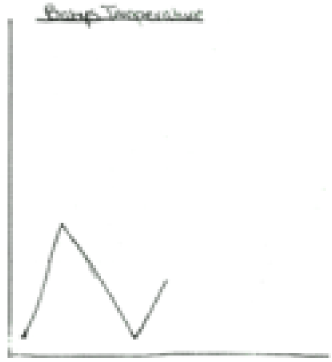
Since I had a small group (six women), I asked if each was willing to draw her sketch on the board. All six agreed. They drew them at the same time. One young woman, after looking at the other sketches, declared herself to be stupid. I told her that there were no right or wrong answers with this; each one had a correct piece of the story. I retold the events of "Giselle's Baby" three more times while tracing the lines of the graphs.

Three sketches were quite similar to each other. They all gradually increased, then dramatically decreased. All three graphs then moved horizontally (or almost horizontally) for a while before increasing sharply again. Another two of the graphs drawn by the students were also similar. Both started with an increase, then a sharp decrease, but these two graphs had no horizontal line (representing the time when the temperature stayed the same). Both graphs showed an immediate increase in temperature right after the decrease. One graph "stood alone." On the last one, we could see that "time had gone backwards" and how the line needed to flow left to right to indicate the passage of time.

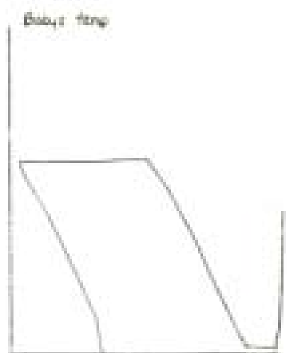
Three sketches were quite similar to each other:



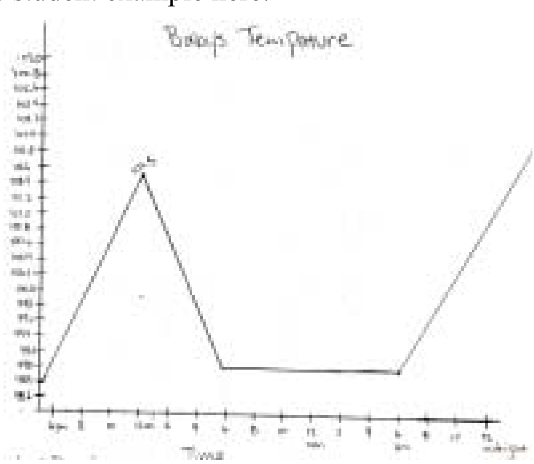
Two more similar sketches:



One sketch "stood alone." On the last one we could see that "time had gone backwards" and how the line needed to flow left to right to indicate the passage of time:



Next, the women took a sheet of graph paper to draw a more precise line graph of the "Giselle's baby" narrative. We discussed where one might want to start the temperature scale and how one might want to count up that scale. We also talked about different ways to label the time axis. The women completed their graphs using the scales they chose and the fever temperatures they chose. They did label the axes and give the graph a title, as seen by the student example here:



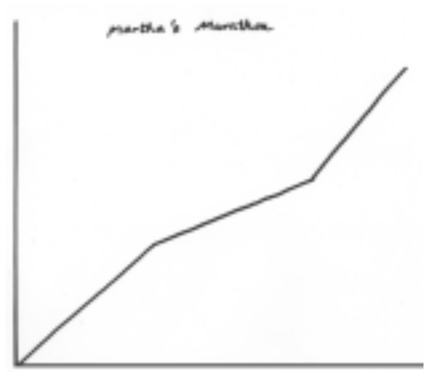
Martha's Marathon

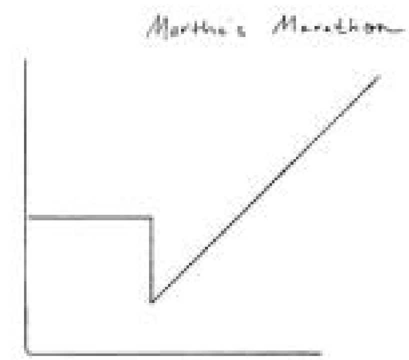
Martha ran a marathon. In the first hour and a half, she ran at a steady pace. Then she slowed down for the next hour, saving her energy for a push in the last half hour when she ran faster than at any other time.

We tried this sketching activity again using the narrative "Martha's Marathon". Each woman agreed to draw her sketch on the board. We retold the story while looking at the sketches.

The sketches were quite comparable except for one. Most students had thought of sketching distance vs. time, and from the start (0,0), the graph slanted upwards (see graph on left, below), leveling off when slowing down, and becoming steeper again when speeding up. Plotting distance vs. time, the line had been a positive slant as Martha increased her distance. But one started out showing a steady pace as a horizontal line (see graph on right, below). She also drew a vertical line to show a decrease. If she were comparing distance and time, a vertical drop would mean that she suddenly lost a good deal of distance. She also did not start at the corner (0, 0). We discussed all of these issues together as a class and found it interesting that the graphs looked different if you were thinking of different aspects of the story. One student was thinking about speed vs. time, and others about distance vs. time. (see graphs below)

From Graphs to Stories





Students were doing math in writing class, and writing in math class.

The students practiced their graphing skills on several other examples before we moved on to reversing the procedure. Now they were asked to choose one of the graphs from the lesson (see box below) and compose a story to go along with the graph sketched. Someone remarked that they were now doing English in math class! Each went right to the task and wrote.

Each student was willing to try this activity as well. Each chose a different context and attempted to use descriptive words to fit the sketch. Students had varying degrees of success with this task. One student used the context of a monthly paycheck. She was able to label the axes of the sketched graph (money and months) and used descriptive words such as "higher", "decreasing", "rapidly increased" and, "another decrease" as well as sequence words such as "after" and "then". Other students created other interesting stories such as the number of customers decreasing and increasing, depending on the day of the week and the availability of strawberries in a store during certain times of the year.

Earlier that morning some of the students had played Scrabble in Language Arts class. They remarked that they had done math in language class as they added up their scores. So, as we proceeded with the math lesson, someone noted that we were doing writing in math class. That tied to a previous day's discussion about math as a language and all the means and symbols we use to communicate in a mathematical way. I hope that they are beginning to see math as a language and that it uses regular communication skills.

Reflection

I believe the lesson was successful in that it was fun, instructive, and engaging. The activities were non-threatening, but informative. Students brought a knowledge of reading line graphs. They also brought knowledge of childcare when it came to numbers for a child's normal temperature and for a fever. One student had run cross-country when in high school. She related to the "Martha's Marathon" narrative.

The initial sketching activities gave students a feel of a line graph without worrying about the precision yet. Also, their sketches improved as the students had more experience. The sketching activity was valuable in that it related an event or information changing over time.

In doing this lesson, students saw that mathematics is not always about being precise. Sometimes a simple sketch is all that is needed to communicate information. My experience with line graphs had been with precise plotting of points. Using a flowing sketch was a stretch for me. I can see its value in helping to identify the flow of activity over time. It helped to consider the vocabulary needed for describing the activity shown in a graph. Since my tendency is toward the math rather than language, it was a good experience for me to have to combine math and communication skills.

Teacher Finds Success Teaching Math by Changing the Order

by Sarah Fearnow

Sarah Fearnow has been teaching adults since 1997 and has been teaching ABE/GED classes at Pima Community College Adult Education (PCCAE) since 2003. The ABE/GED students at PCCAE are a diverse population ranging from the ages of 16-75. Often about half of the students in a given class speak Spanish as their first language and English as their second language. The classes are taught in four 10-week sessions that run throughout the year. Students are enrolled into the classes at the beginning of each session and at one time during the session. PCCAE offers two levels of classes. Students scoring at an 8th grade equivalency level or above on the TABE test are placed in the GED class. Students scoring below an 8th grade equivalency level on the TABE test are placed in an ABE class. Sarah teaches both of these levels of classes at different times during the day. Math class is offered three hours a day, twice a week.

Developing Math Content Strands at All Levels challenges the traditional scope and sequence.

Since learning about the TIAN philosophy towards teaching math to adults, my teaching strategies have totally changed. One major change is the order in which I present the mathematical concepts to students. Previously, I had always taught mathematical concepts in the order they are presented in the text books: multiplication and division before fractions, decimals, and percents, and the latter three before algebra and geometry. I held off teaching algebra and geometry until students were testing at above an 8th grade equivalency level and thoroughly knew the basics.

The TIAN institutes gave me the seemingly radical idea of teaching the concepts "out of order". Now, I teach both basic algebra and basic geometry to my ABE students. As a result of this simple change, I have had better retention of my lowest level students. Before, when I had the students studying multiplication and division exclusively before moving on to other topics, they would seem bored and frustrated doing page after page of multiplication drills. I think they felt like they were spinning their wheels and were uninspired and unmotivated to stick with it long enough to actually learn the times table and move ahead. Now I feel the students stay in class longer because they feel like they are making real progress and are getting closer to being ready for the GED. Now, my students appear more interested in class and actually find math class fun.

This past class session, I discovered that my ABE students didn't know the

Teaching in context with real applications helps students understand when to multiply and when to divide.

multiplication table. We discussed how being able to multiply and divide quickly would help them pass the timed-GED Math test in the time allotted. We decided to spend three months focusing entirely on multiplication and division. Instead of simply drilling the students on the times table all session, I introduced the students to some areas of math that require using multiplication and division, including area and volume in Geometry and In and Out tables in Algebra, and in this way, the students practiced multiplying and dividing in applied contexts and had fun doing it.

One way we practiced the times table was by doing area and volume problems such as the examples on the right. When finding the area or volume of any shape, you must multiply; for example, the area of a rectangle is length times width, the area of a parallelogram is base times height, and the area of a triangle is one half base times height. If the area and one dimension of a shape is given, you must divide to find the missing dimension. We practiced finding the area or a missing dimension of various shapes of differing sizes and thus, were able to practice multiplying and dividing over and over again in a meaningful context, instead of simply doing drills that seemingly had no meaning.

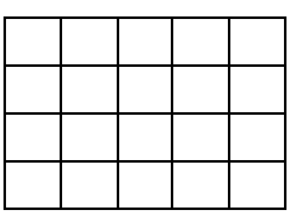
> What is the area of a rectangular garden that is 6 feet long and 15 feet wide?
>
> What is the length of a square garden that has an area of 144 square feet?
>
> What is the area of a parallelogram that has a base of 10 inches and a height of 12 inches?
>
> If the area of a parallelogram is 48 square cm, and the base is 6 cm, what is its height?

The teacher explicitly clarifies what they are learning and why.

Throughout the activities, I continually reminded the class that our focus was to learn the times table and the process for long division because knowing those things would assist them immensely on being able to finish the GED test within the time given.

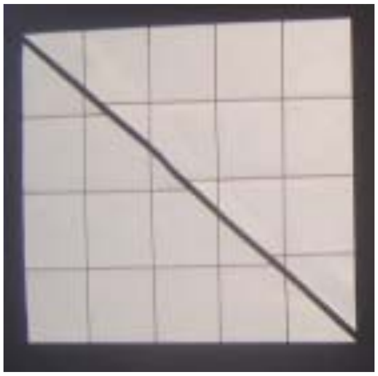
Hands-on exploration of formulas takes away the mystery.

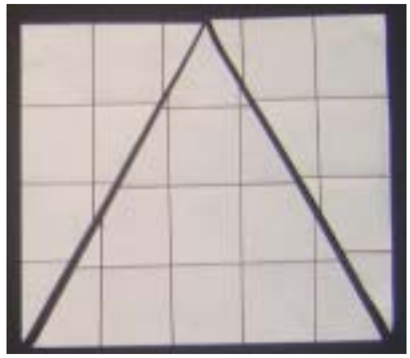
Shortly into the unit on area, it occurred to me that my ABE students were using the formulas, but not understanding how the formulas had been derived. I wanted my students to gain a true understanding of the area formulas, not just to be able to use them, so I led them through a hands-on activity that ended up being very successful in building understanding of the formulas. We created various shapes out of paper and compared and contrasted the areas of each. We started by creating a 4x5 grid pattern on a rectangular piece of paper. We then discussed how we could see the area of the rectangle according to the grid we had produced (there were 20 squares on the paper) and why multiplying length (5 units) times width (4 units) gave the area (20 square units).



Using a grid to see multiplication.

Then each student created three more identical rectangles. We cut two of the rectangles into two triangles each (two right triangles and two non-right isosceles triangles). After creating each pair of triangles, I asked the class to prove to me that the area of the triangles was exactly half the area of the original rectangle that the triangles were cut from – in other words, 10 square units. Each time, the students eventually came up with the idea of laying the triangles on top of each other to show that they were identical in size and shape and therefore consisted of exactly half the total amount of paper. The students proved that, even though the triangles we made were shaped differently, they still both had an area that was exactly half of the area of the original rectangle and could deduce that the same idea would hold true for any triangle of any shape.

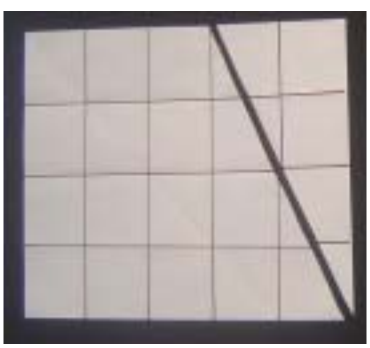




Prior to doing this activity, my students had expressed confusion about why the formula for the area of a triangle (area = ½ base x height) included the ½. They didn't feel comfortable with fractions in general and felt thrown off by a fraction in the area formula. After doing the activity, the concept of the triangle being half the rectangle made a lot of sense to them and they felt more comfortable using the formula.

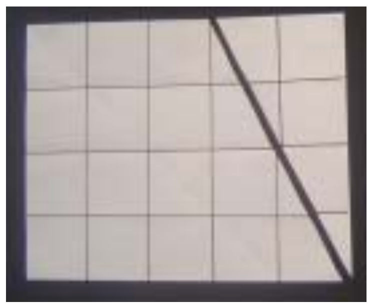
A combination of hands-on and visual representations encourages students to use different modalities.

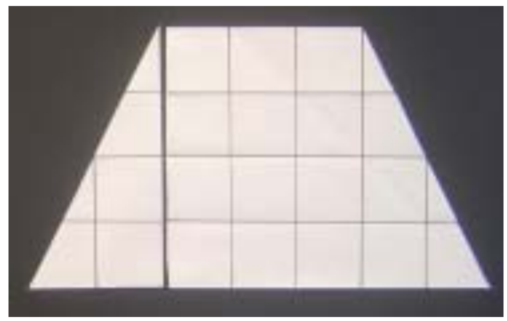
With the last rectangle, we created a parallelogram and saw how the area was identical to the area of the original rectangle (20 square units), not half the area, as it had been with the triangles, because it took the whole paper to make the parallelogram.



The students looked at the area formula for a parallelogram (area = base times height) and discovered that "base times height" is the same as "length times width" – the dimensions used in the area formula for a rectangle. In this way, the students could see that the formulas were essentially identical. Because they saw that "height" in the parallelogram formula was the same as "width" in the rectangle formula, the students understood why they had to use a vertical line depicting the height of the parallelogram in the area formula, rather than the diagonal line around the perimeter of the parallelogram. This activity seemed to make the area formula for a parallelogram much less confusing for my students.

We were also able to create a trapezoid with the same paper we used to make the parallelogram. We saw that the trapezoid we made used the whole paper, and thus, as with the parallelogram, it had the same area as the rectangle (20 square units). We then explored how the formula for the area of a trapezoid ($A = ½ (b_1 + b_2)h$) was derived. The students remembered from the parallelogram that the height meant the same thing as the width in the area of a rectangle formula. They deduced that the rest of the formula for the trapezoid ($½ (b_1 + b_2)$ must equal the length of the original rectangle. We discovered that $b_1 + b_2$ results in a number that is twice the length of the original rectangle. Then it became clear why we had to divide that number by two to get the correct area (20 square units).





The students loved this activity because they really enjoyed the hands-on aspect of it and because they felt it made the area formulas much more accessible to them.

Once students had gained a solid understanding of the formulas and had practiced using them in various contexts, I gave the class a project of applying the formulas to a real-life situation: pretending to redecorate our classroom. In groups, they were assigned to figure out how much carpet or wall paint would be needed to redecorate our classroom. The idea for this activity I took directly from the *Over, Around, and Within* EMPower book.

***All* students participate, *all* students use multiplication and division and geometry and formulas, but at different levels.**

For my more advanced students, I brought in a packing box and had them find the volume of the box. Then I asked them, If our classroom were turned into a storage room, how many of those same sized boxes could fit in the room? This task involved dividing the volume of the room by the volume of the box.

I also had the advanced students find the area and volume of the trash can outside our building (see picture on the right). I gave the groups tape measures and asked them how much stuff the can could hold, how much material would be needed to cover the cylindrical side of the trash can, and how much metal was used for the lid, which has a hole in it for tossing trash in. This last piece requires finding the area of two circles and subtracting them.

For all of these applied math activities, my students loved the chance to get out of their seats, the hands-on aspect of using the tape measures, the challenge of a real-life problem with real-life applications, and the opportunity to work together, which always makes class more fun. I loved how each person in a given group had different strengths they could bring to the project and how the students were really learning from each other in this activity. I am grateful that the TIAN Institutes gave me the idea to assign real -life projects to my students to apply the mathematical concepts they had been learning in class. This type of project-learning brings math to life for my students.

Making math real: Understanding why students need to learn different concepts

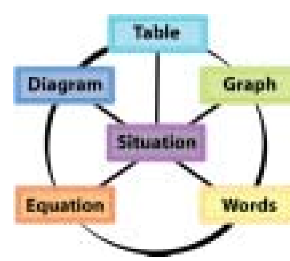
Another application of multiplication and division I taught, in addition to the area and volume formulas, was in Algebra. I learned in the TIAN Institutes that part of Algebra is recognizing patterns and that one way to practice recognizing patterns is by identifying the function applied in an In and Out table. We looked at tables that involved multiplication and division in order to become very familiar with seeing the patterns in numbers. I suggested to my students that learning to see patterns in numbers in this way would help them see patterns in the times table and thus, help them be able to memorize the table more easily. We finished up this piece by looking at In and Out tables that involved both multiplication and addition. In the example on the right, the rule for the table is y = 5x (to get the value for y, multiply the value for x times 5).

Using algebra to reinforce basic multiplication facts

In and Out table

x	y
6	30
7	35
8	40

I always teach Algebra now using the wheel that we saw in the TIAN Institutes, where you connect the ideas of a real-life situation, the graph, the equation, and the In and Out table. My students really respond well to learning Algebra this way, especially students who had struggled with Algebra in the past. So, throughout the lessons of looking at In and Out tables, I helped my students discover how the tables could be connected to a real-life situation and an equation, and we created the corresponding graph.



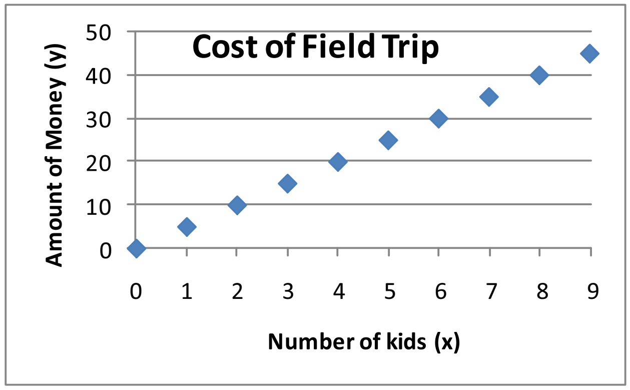
For example, one student connected the idea of the above In and Out table to the situation at her son's school. She said that parents must pay $5 per child if the class goes on a field trip. Her new In and Out Table on the right illustrates the total amount of money that must be collected (the y values) for varying numbers of kids attending the field trip (the x values). While it doesn't show up in her table, she realized that someone who had no children would pay nothing. So, when x = 0, so does y.

Make algebra come alive using real-life contexts familiar to the students.

Field Trip Costs

x	y
1	5
4	20
6	30

The corresponding graph that the students created did include the point (0, 0) and, if the points were connected, the graph would be a straight line. The students' graph looked similar to this:



The connection to real-life situations really brought the activity to life and the visual representation of the situation in a graph form also made it more tangible, more accessible, and more related to what the students might see on the GED test. After inventing situations that could be related to the tables, we flipped it around and started with real-life situations and then created the tables, graphs, and equations that could correspond to them.

In-out tables, graphs, and even equations can be developed from real -life scenarios.

One example of a situation we used was the idea of planning a child's birthday party at Pump-It-Up, an indoor jumping castle place (www.pumpitupparty.com). I researched the party package prices online and gave them to my students with the task of planning a party for 20 kids and then for *x*-number of kids. The package price includes a $250 flat fee for up to 25 kids and then costs $10 per additional child. A parent also has the option of buying one goodie bag per child and buying pizza by the pie or by the slice. This allowed for a lot of flexibility and variety in planning party scenarios. So for example, for a party with $4 goodie bags for each of *x*-number of kids, the equation would be Total Cost = $250 + $4x, where *x* represents the number of kids attending the party if $x \leq 25$. Most of my students have children and have planned birthday parties for them before, so this activity was familiar to them and relevant to their lives. That made the mental leap of thinking about the situation in an abstract way (inviting *x* kids) a little easier. My class had a lot of fun with this activity and I was impressed with how easily they were able to practice algebraic thinking in this real-life context. And once again, they were required to multiply every time they included a cost per child in their party scenario.

Development and integration of math content strands at all levels also helps students make important connections.

I am so glad that the TIAN institutes gave me the idea to teach the math topics "out of order" because my students have greatly benefitted from the changes I have made in my class. I believe all of my students now feel like they are making progress towards reaching their goal of preparing for the GED. They also make connections and see how topics in mathematics are interrelated. Being involved in TIAN has made me a much better teacher because I am able to make math fun, meaningful, and accessible to a greater number of people than before. My students tell me they enjoy my math class more now because they are able to work in groups, teach each other, and see the real-life applications of concepts we are studying in class.

Listening to Students
by Lynn Foley

Lynn Foley has over 20 years of experience in adult education. She has worked as a GED teacher for Project RIRAL (Rhode Island Regional Adult Learning) since 1996 and for the past seven years has been teaching out of school youth, ages 16-24. Her class has a somewhat open enrollment policy. Students are allowed into the program at the beginning of each month from September to May. RIRAL also runs a 7-week summer program for the same students. Her story is about students attending the summer program. Lynn shares, "I struggled with math throughout my school years and so have a special interest in helping my students overcome their problems with it. I try to make it as painless as possible and am often told that if their middle and high school math teachers had just showed them how to do math the way that I show them, they could have 'gotten it' a long time ago. I do a lot of hands-on activities and draw a lot of pictures! I always tell my students, you just need good 'strategies' (counting on your fingers, drawing pictures, making tally marks on your paper, etc.) to be good at math!"

My class is set in a community-based program in a small inner city in Rhode Island. The students are all low income youth ages 16-24. Placement in my class is based on a reading CASAS score of at least 226 on the pretest. I don't worry too much about their math scores, allowing any score into the class. So, I have students with math CASAS scores that range as high as 245+ and as low as 200. With the exception of one student, everyone has stated a strong dislike for math!

This summer I have a class that meets two days a week from 9 am to 3 pm both days. The students have asked for math in the morning, from 9 am to 11am, reading after lunch from noon to 2 pm, and a chance to do independent study from 2 pm to 3 pm. There has been an open enrollment policy for the first four weeks of the class to achieve the cap of 20 students enrolled.

We started working with proportions during the first week of class. I introduced the topic with various examples of ratios and proportions in everyday life. We looked at supermarket ads, gas prices, measurement/recipe conversions, etc. The snippets I'm sharing here are about different students' reasoning with proportions.

Lucy's Thinking about 2 ⅓ Hours

Today in class, our first day of summer school, a student was trying to figure out how many minutes were equal to 2 1/3 hours. She knew that 1 hour was equal to 60 minutes, but didn't know where to go from there. I asked her what she was thinking. She said because the ⅓ was there, she didn't know what to do next. I told her to ignore it then; do the problem without the ⅓ first. She told me she was determined to "get fractions" so didn't want to skip it…she wanted to know how to do it with the ⅓.

How many minutes are in 2 1/3 hours?

Adult learners come with prior knowledge which teachers can use to build new knowledge and make connections.

I am a very visual learner and therefore a very visual teacher. I draw pictures and diagrams for everything. Nothing says math like a picture! Another thing I try to do is not use "math terms" when trying to get students to understand a new concept; I try and keep it simple using simple words that they can hopefully use their prior knowledge to relate to and then make the connection to math themselves. But, it all depends on the level of the student I'm working with. Lucy is very frustrated with math and working at a low level. She has taken and passed all her GED tests, except the math. She scored a 300 on the math GED test.

Communication can consist of the teacher explaining to students; another form of communication is the teacher asking questions to encourage students to figure out their own thinking.

I asked Lucy to show or explain to me what her understanding of ⅓ was… whatever it was. She said she knew that it meant that one part was shaded in.

"Can you show me that in a picture?" I asked.

"I don't need to; I know that one part would be shaded in."

"One part of how many would you shade in?"

She wasn't buying in to the picture scene! And I was gathering that she wasn't sure what she meant by "one part is shaded in"; she had been being helped by a student who has been with me since September and I think this was her contribution to the teaching of fractions—showing Lucy a picture of something divided into three parts with one part shaded in.

At this point Lucy worked with the two hours to determine that they totaled 120 minutes. She completed this by writing on her paper: 60 + 60 = 120.

"Is your final answer going to be more than 120 minutes?"

"Yes, obviously."

"Why is that obvious?"

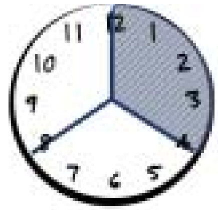
"Because 120 is just the two hours, there's still more that I have to add into that answer."

"Right", I said, "you still have to add in the other one third of an hour."

At this point, a light went on because her eyes flew open and she said that she hadn't known that that was what the ⅓ meant. That threw me a curve and I asked her to explain what she meant!

"I think that just means that I have to take an hour, which is 60 minutes and divide it by three and then add that to 120. I know how to divide!" At which point she took pencil to paper and wrote a division problem that showed 60 divided by three!

I felt compelled to draw her a picture to show her what I was talking about, so I drew a circle (clock) and divided it into three equal pieces asking her to tell me how many minutes were in each portion of my clock. She then counted by 5's to get to the answer of 20 minutes in each section. Then I made the connection between the "one part shaded in" peer lesson and what she did with the division. She said she got it now but saw no need to draw it out….she would just divide it – it was much easier! I pushed her and asked what if the problem had said ⅔ of an hour instead? She first thought she should then divide it by two but quickly determined that wasn't right…it didn't make sense. I left her with the problem and went to work with another student.



How many minutes in 1/3 hour?

Pushing on building conceptual understanding rather than just procedural fluency

Not long after, the other student working near Lucy yelled, "She's drawing it out in a picture!!" I asked her to please explain what and why!!

"I can't figure out what the heck to divide, so instead I drew a stupid picture like you did and made it (the circle/clock) into three sections. I knew that one section was the 20 minutes that I had already added, so then another section had to be 20 minutes so I just added that too. Wait! I get it…I could have divided it by three still (60/3) and just added 20 two times instead of once cause it's 2 thirds! I get it!"

The student uses adaptive reasoning to figure out how to deal with a situation and then is able to explain and justify her reasoning.

Benjamin's Tentative Proportional Reasoning
Benjamin is a low-level math and reading student who struggles with ADHD issues and very low self-esteem. He doesn't like to work in groups for fear of exposing his deficits in math.

The math activity that was assigned to the class was to plan a fundraising event that would involve figuring out how long it would take to stuff 1000 envelopes and decorate 100 tablecloths each measuring 10' x 4'. These activities centered on the math topic of proportions.

I insisted that the class break into groups of three to start this assignment. Benjamin reluctantly went with two other students to figure out how long it would take to stuff the envelopes. As a group, they decided to stuff one envelope and time how long it would take. They came up with ten seconds per envelope. I let them be. I didn't want to push them to take a bigger sample for fear of losing the "math" in the activity. They liked their sample and were ready to work with it.

Benjamin started listening to the group's plan of writing out proportions of 1/10 = 10/100 = 100/1000 but was getting frustrated with their logic. He shut down and told me he couldn't do it…he didn't know that kind of math. I asked him to do something to let me know where he was, what he DID understand about the timing-of-envelope exercise in which he just took part.

Teacher begins with where the student is—questioning what he does know about the situation at hand.

"I know that it took me ten seconds to stuff one envelope." He said this as he drew it out on paper.

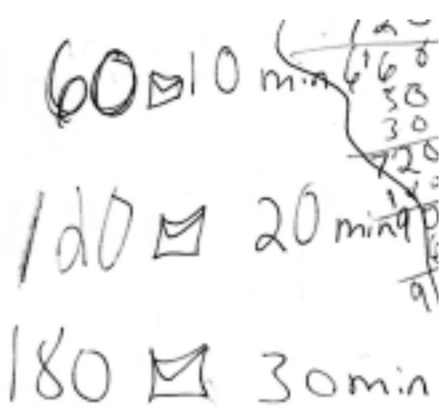
"OK, then what about if you were going to stuff more than just one envelope, can you show me that?"

He then continued his "list" showing me that it would take 20 seconds for 2, and 30 seconds for 3, etc. I believe that Benjamin could not make that connection if he didn't write it down the exact way he did.

Student's understanding and use of proportional reasoning is a developmental process, moving from additive to multiplicative reasoning. The teacher allows for this development by allowing student to build a table and look for patterns.

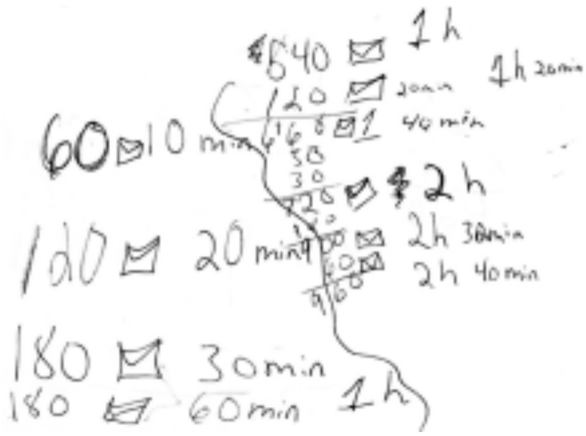
Benjamin got to 60 seconds for 6 envelopes and started to continue, but then hesitated and it looked like it "dawned" on him that 60 seconds = 1 minute, so he continued on the second column of his paper and changed his pattern to be 6 envelopes took 1 minute. He continued with the "proportional" reasoning chart increasing by one minute each time until he got to 30 envelopes in five minutes

Then, without hesitating , he jumped to doubling his chart to show 60 envelopes in 10 minutes and then doubling it again to 120/20. He went to 30 minutes at this point, so I asked him why. "I just added those two together (referring to the 60/10 and the 120/20 equaling 180/30.) Then I'll just add another 30 minutes and another 180 envelopes and that will be an hour."



At this point I told him he was doing a great job and to continue on with the "goal" of figuring out how long it would take to stuff 1000 envelopes. I went to work with another student.

Upon returning to Benjamin, I asked him to explain to me where he was. "I figured this many envelopes in one hour." (He pointed to the 540 on his paper) (This is where he went wrong. He added the wrong things…for some reason it looks like he added 60+120+180+180 to equal what he thought would take an hour to stuff, coming up with the 540. But, when I asked him how he got that 540/1, (I, too pointed at the number 540 instead of saying it), he told me that he added the two total envelopes for the two 30 minutes. I didn't stop him at this point for a couple of reasons: 1) he was on a roll and excited about his reasoning, and 2) he was telling me the "right" thing to do, but just didn't do it at the time. I felt I could go back later and talk to him about his mistakes.)



"Then I took the 20 minute envelopes (120) and added that and the minutes. (He did this addition correctly coming up with 660 envelopes in an hour and 20 minutes, BUT labeled it on the side of the 20 minutes and labeled the 660 with 1 hour and 40 minutes!) At this point, he continued to add the envelopes correctly, but got thrown off with the minutes. He explained to me that he was trying to get as close to the 1000 envelopes without going over. I think his biggest problem was organizational – not mathematical!

Sometimes listening all the way through a student's thinking is more important than interrupting his explanation to correct him.

Tracy's Reasoning
During the second week of class my husband and I drove to Wisconsin. During the trip, I kept track of various aspects of the trip that I thought the students would find interesting for their proportions lessons.

To start the third week of class, I presented students with the following list: I then had them divide themselves into groups. This started out with 3-4 students per group with four groups. (During the fourth week, six new students joined the class and were either "invited" to join an existing group or formed their own group. Each group welcomed the new members and

<u>Facts about my trip to Milwaukee, Wisconsin</u>

- It took 44 hours to drive to Milwaukee, Wisconsin and back to Massachusetts for a total of 2200 miles.
- We used 220 gallons of gasoline to make the total drive.
- The average cost of gasoline was $2.75 a gallon.
- The most we paid for gas was $3.00 a gallon in Indiana and the least we paid for gas was $2.60, also in Indiana.
- We drove a total of 111 miles through Massachusetts, 401 miles through New York, 54 through Pennsylvania, 252 through Ohio, 169 through Indiana, 73 through Illinois, and 40 miles into Wisconsin.
- Massachusetts tolls totaled $8.70, New York costs $58.75 Pennsylvania was free, Ohio costs $35.75, Indiana costs $49.90, Illinois costs $28.20 and Wisconsin was free.
- On my map, one inch equals approximately 100 miles.
- For one night in the campground, it costs $28.00.
- I drank 42 cups of tea, stopped to go to the bathroom 126 times, ate five boxes of Cheez-its, and gained 15 pounds!

Teacher addresses fluctuating attendance by assimilating new students into existing activity.

brought them up to speed with the assignment.)

The assignment was:

1. As a group, brainstorm questions surrounding the facts of my trip. (this was expanded upon and clarified by modeling some on the board as a whole class)
2. You should have at least ten questions per group.
3. Divide the questions between group members and solve independently. Each group member should solve at least two questions independently.
4. Explain/show your method for solving each question by using pictures, written explanations, and/or proportions.

Finally, each group came up with a set of questions. The following is a sampling of the questions the students generated.

Once the questions were completed, they were asked to solve them independently. Some groups insisted on continuing to work together, but said they would "show" their individual work for two problems using

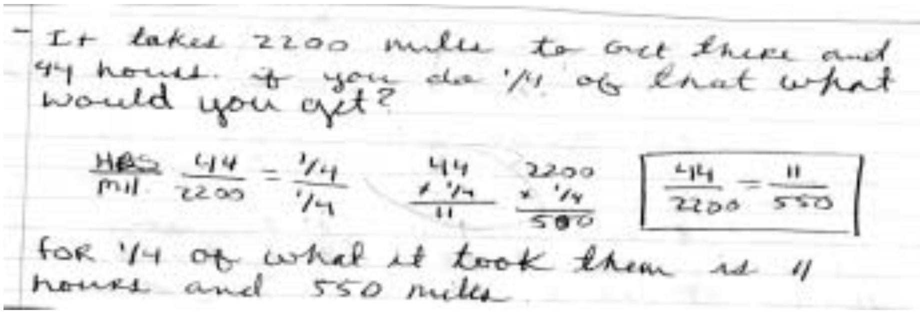
Student Generated Questions

1. How many miles, on average, did she travel between bathroom stops?
2. If it takes 44 hours to drive 2200 miles, then how many miles did she drive in one hour?
3. It takes 2200 miles to get there and 44 hours. If you do ¼ of that, what would you get?
4. If $2.75 is the cost of one gallon, then how much would it cost for two hundred and twenty gallons?
5. If $605 is the cost of 220 gallons and $2.75 is the cost of one gallon, then how much is the cost of half of 220 gallons?
6. If it takes 401 miles to go through New York, then how many miles would it be going half way through NY?
7. On a map, one inch is 100 miles. So, if you drive 2200 miles how many inches is there?
8. How many miles did they drive in 10 hours?
9. How many miles did she drive using 5 gallons?
10. How many gallons did she use if she drove 100 miles?
11. How many miles would count for 7 inches on a map?
12. How much would it cost to stay in the campground for 6 days?
13. If she drank 84 cups of tea how many times would she go to the bathroom?
14. If you got 10 miles to the gallon, how many gallons were used through Indiana?
15. If she ate twice the amount of Cheez-its, how much weight would she gain?
16. How much would it cost to drive half the distance?
17. How many miles would it be to go ¼ of the trips distance?
18. If there are 2 people sharing the campground cost and the cost for one night is $28, between the two people how much would each person pay?

pictures and written explanation.

Going over their completed packets, I asked them to meet with me individually to discuss some of their work. I called Tracy up to explain one of the problems she did.

Students communicate with each other but also are expected to communicate individually with the teacher.



Students are encouraged to use their own strategies to solve problems.

"I helped the rest of the group with that one, but that's not my actually problem," she said.

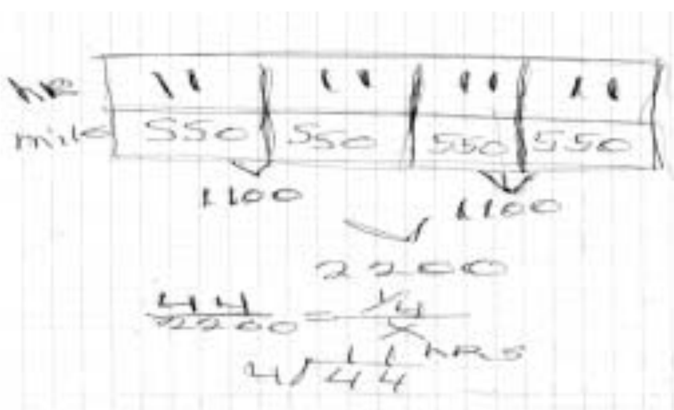
I have two Tracy's in the group and called the wrong one up! Because she said she "helped the rest of the group" with this problem, I asked her to explain what the other Tracy had done.

Student uses adaptive reasoning to make sense of how she and another student's strategy.

"I don't know why she put ¼ over ¼ and then multiplied 44 and ¼ and 2200 and ¼."

I asked her if she could tell me if the answer was correct – the 11/550. She started to draw it out, explaining as she went: "I'm going to draw it out like you do. If we want to know what one quarter of the trip was, then we have to divide the trip into four pieces. If 550 is right, then the four of them would add up to 2200."

She proceeded to put 550 into each of the four sections she had and then added them up 2 by 2. "550 plus 550 is 1100, and 1100 and 1100 is 2200 and that's the total trip, so yeah, she did get it right. And yeah, 44 hours divided by 4 is 11 each, so that's right too. So to answer the whole question, one fourth of the trip is 11 hours and 550 miles. She multiplied, but I divided. That's cool to know you can do either in that situation! I'm just better at seeing it in division!"



Lucy's Thinking about half of 401 miles
The next student I had a question for was Lucy (remember Lucy from earlier in this story?). She is still struggling with proportional reasoning and tends to go with the group. She did pass in completed work, but I wasn't sure she understood what she, or the group, had done.

Questioning so the student has to figure out his/her own reasoning is critical to building mathematical proficiency.

"Lucy, I can see that you labeled your work with miles on top and per on the bottom. That's always a good start, to label things to keep them straight. What did you mean by per?

"I put 401 on the same level as the miles because that was the total miles through NY. I don't remember why I put per."

"Ok, how about the 100. What does that represent?"

"I don't know."

I could tell she was getting worried and insecure. I didn't want her to shut down, so I quickly told her her answer was correct, that 200 ½ was the distance half way through NY.

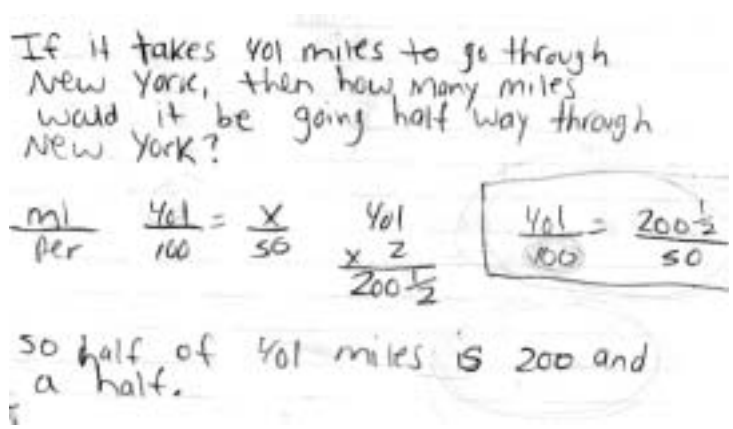
"I did? I got it right?"

Teacher helps students develop adaptive reasoning by asking meaningful questions that ask students to explain their reasoning and justify their answers.

"Yup, but now I want you to show me how YOU would get it."

"Ok, first I thought that 400 divided in half is 200. Then I figured half of 1 is half, so half of 401 is 200 and 1/2. Because 200 ½ plus 200 ½ is 401"

She was very excited that she was able to tell me how SHE got it but was even more excited that she proved it to me, too! I wrote down what she said as she said it. She finished her explanation with, "So you times 401 times 2." (UGH) I wrote that multiplication problem down and asked her to do the multiplication with me. When she saw that that problem equals 802, she got confused again. I showed her the sentence that I had written down when she was explaining it to me – *400 divided in half is 200* – and I underlined the word divided. I wrote out that problem and asked her to solve that one with me too.



"I don't think I should times it. I think I did times because I'm better at multiplication than I am at division and I saw the x on the problem and thought that meant multiplication."

Having asked the student to explain her reasoning, the teacher now knows why the student used multiplication instead of division.

Close

Since being involved in the TIAN training for the state of Rhode Island, I have embraced the TIAN model. I feel that asking adult education students questions about their math thinking is vital for helping them through their journey from GED to college or career. Most of these students haven't had any success at "school math," especially the youth I service, the 16-24 year olds. They're often intimidated, embarrassed, and frustrated with their lack of basic skills.

To have a teacher take the time and ask them to explain, or show, what they ***do*** know can be empowering for the student and enlightening for the teacher. I am always surprised at what my students ***can*** do even though they have failed every other math class they have taken - even though they don't know their multiplication facts and believe they will never "get" fractions. If you delve a little bit deeper into their thoughts by asking questions instead of just teaching drills and facts, you will better understand ***their*** logic and be able to build on what they do know so they can be successful!

Does It Have to Go Up to 600?
by Holly Lee

Holly Lee is an ABLE teacher at Great Oaks in Cincinnati, Ohio. She teaches at two different sites, one urban and one rural, but both with open entry classes where students work on their GED or review for college or career entrance testing. Students range in ages from 18 to over 80. Holly has been teaching in the early childhood field of education for 21 years and adult education in the evenings for 19 years. Holly sums up her love of teaching by saying, "I go to work thinking that I am going to teach, and more often than not, I come home realizing that I am the one who learned."

I teach math. I enjoy teaching math. It still surprises me to see those words in print. Teaching math to adults in the ABLE (Adult Basic Literacy Education) program in Ohio was not on my original career path; however, I'm glad that I took not only the opportunity, but also the risk. I would have missed out on the most rewarding experience of my life.

Not only do students need to have a productive disposition, but so do teachers.

In 2005, I was part of the TIAN pilot project. The TIAN philosophy of discovery, small group work, confidence building and using "real life" experience models appealed to the creative side of me. I came away from the project determined to find more and better ways to make math not only attainable (as in passing the GED Test), but also fun (as in, "Who knew you could laugh in math class?!"). Fast forward five years and I'm still at it.

Building a common ground encourages communication—and commitment to attendance.

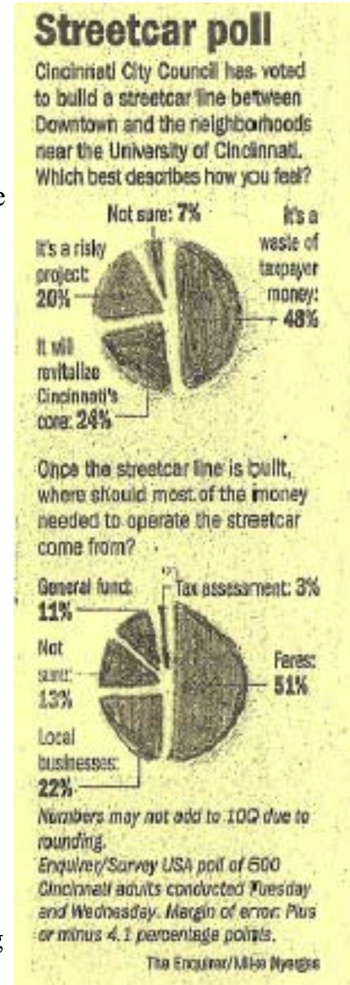
My classroom attendance is never the same twice. The program is voluntary and students can come and go as they please, so the first challenge becomes to create a place of acceptance and success so that they want to be there. A good way to do this is to try to find some common ground. Generally, the first common ground everyone agrees on is: "WE HATE MATH!" Not exactly the best start, but it is a start.

WorkKeys is a job skills assessment program used in many workplace programs.

The group experience I'm going to share here centers around a core group of five women who range in age from mid 20s to about mid 50s. Some of them are working on their GED, another is focusing on the WorkKeys Test. There are often more students in class, but these five are the most consistent.

One of the women, Stacie, had been working at home in a chart/graph/table textbook and was having difficulty understanding what was expected of her. She wailed, "What do they want me to do?" Her classmates were quick to jump in, sharing their own frustrations and *voila*, we had common ground.

The next day, in our local newspaper, my lesson plan was provided. Our city is embroiled in a controversy about building (and paying) for a streetcar line between downtown and the university area. (It's certainly not a life or death situation, but people seem to have strong opinions about it.) There in black and white were two lovely circle graphs complete with percentages, titles, a hidden "whole" and all sorts of great math stuff. My TIAN experience helped me realize the importance of "real life" connections, so the graph/chart/table textbook went out the window and we went to work on a *real* graph.



We started by examining the information. I asked them, "What do you see?" Responses were: "a streetcar poll", "circle graph", and "how the people of Cincinnati feel about a streetcar poll in percents".

Posing the right questions can be very productive. The teacher illustrates powerful communication by asking simple questions that force students to think critically. Rather than give, or hint at the answer, she nudges students to reflect critically.

Then I asked them, "Where do you see math?" Responses included: "math is in the circle graph", "percents", and "4.1 percentage".

The next question was, "Where did these numbers come from?" The immediate answer was, "the mayor" and "city council". When I asked them to show me where they found that information, they decided that they were wrong and that the numbers came from all the voters in Cincinnati. "Really?" I responded, "I vote and nobody asked me about this. My feelings are hurt." Someone said, "Wait a minute! I know! They asked 600 people. It's down there at the bottom of the graphs!" At that point, everyone started talking and I heard things like, "Oh, I get it now – 48% of six hundred people think it's a waste of money" and "six hundred is the whole" and, "Is 600 the whole for both graphs or just the top one?" HALLELUJAH! We have lift off!

What people said	Percent	Part fraction "piece of the pie"	Whole total "how many people were asked"	Estimate "mental guess"	Set up for the problem "the math"	Actual number of people the "real number the actual answer
Waste of $	48%	Is about half	600	A little less than 300	$\frac{?}{600} = \frac{48}{100}$	288
Revitalize core	24%	About ¼ ½ of ½	600	About 150	$\frac{?}{600} = \frac{24}{100}$	144
Risky project	20%	Less than ¼	600	Around 130	$\frac{?}{600} = \frac{20}{100}$	120
Not sure	7%	A lot less than ¼, s Small number	600	Maybe 30	$\frac{?}{600} = \frac{7}{100}$	42

Before the next class session, I created a chart on the whiteboard that listed all of the information that the students had discovered during the previous class session, but deliberately left off the titles/column headings across the first row. The first group activity was to examine the chart , and then title each section (see completed chart on previous page). This review was important because I do not see my students on consecutive days and often there are new students attending class who need a "frame of reference" if we are in the middle of an activity.

Teacher manages turbulent attendance by using a review strategy.

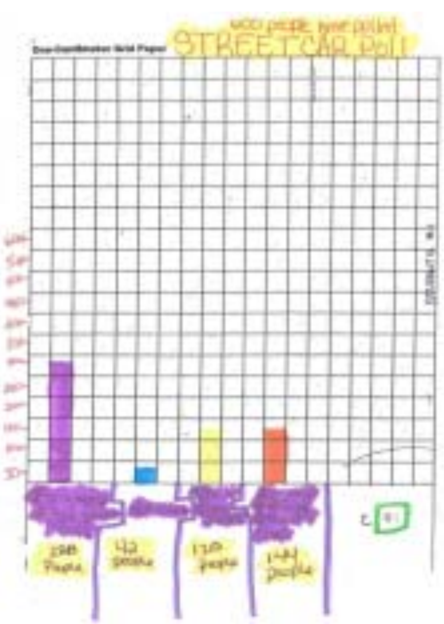
After everyone was "refreshed" on the data, I provided graph/grid paper and different colored markers and asked each person to create a bar graph that would reflect the information from the top circle graph. Even though they had seen bar graphs in textbooks, they were not completely sure how to go about making one. It was at this point that I tried to take a step back and let their own problem solving skills develop. It would have been very easy for me to "guide" this activity and end up with all the graphs looking exactly alike, but then the element of discovery would have been lost. No one began immediately; they all spent some time considering the task.

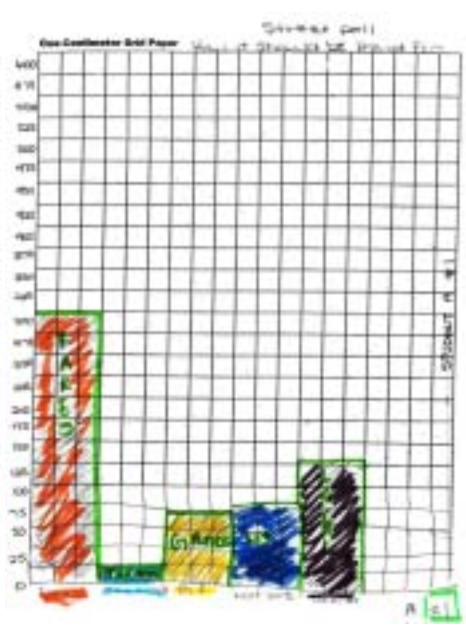
Connecting bar graphs and circles graphs helps to build conceptual understanding.

Each person had a different strategy for getting started. One student started with a title. Two others started by labeling across the bottom of the paper (x-axis) and one started listing numbers going up the left side (y-axis). One student simply sat and observed the others. The student who started with the y-axis quickly got up and got another piece of paper, explaining, "The numbers I chose are not going to fit. I was going to go up by tens but the paper isn't big enough to get to six hundred."

The minute she said "six hundred", that seemed to jumpstart the others. The structure of a bar graph began to take on a recognizable form with that comment. As they worked, they would often look at each others' graphs and it became apparent to them that although they were all working from the same data, the "pictures" were not turning out the same.

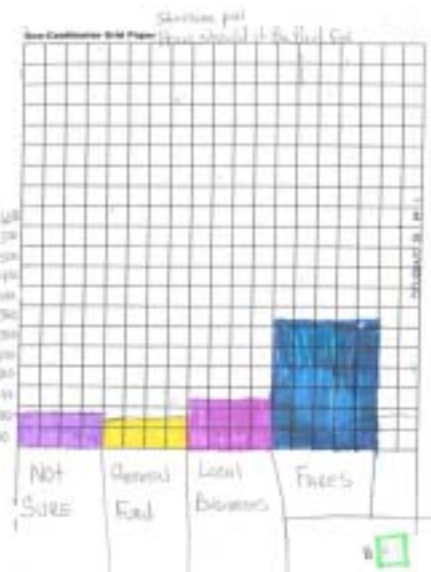
Students learn from each other as they communicate freely with one another.

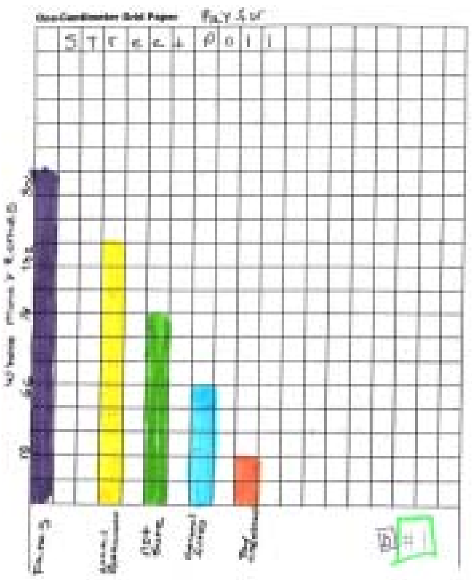




Students communicate their understanding as they develop adaptive reasoning.

After they were finished, we put them all together to compare. The most discussion centered around the size of the bars. Two graphs had bars that were only one grid-square wide and two graphs had very “fat” bars. When questioned, Student B, who chose to make wide bars (graph on the left below), explained, “What’s important is the way the bars move up and down, not sideways. As long as I go up the right amount, it doesn’t matter how fat I make the bars. I tried to fill up the whole page.”





Then Student C, who used single blocks as the bars (graph on the left on the previous page), said, “Oh, yeah, on mine there are spaces between the bars. They’re kind of spread out. I can see that if I colored in the white spaces ours would look a lot alike.”

Student D (graph on the right above) did not move the y-axis in equal increments. She simply labeled it using the data amounts. She knew that numbers belonged on that side, but did not “get” why equal increments were important, especially since her graph looked much like the rest. (This is also the student who asked to work on graphs at the beginning of this story.)

Interestingly, Student E (we’ll call her Sharon), who had been observing all this time, was still observing and that was okay with me. People are different. In my head, I compare math to swimming. There are those who jump in the pool with little or no regard to depth or temperature, others who ease in gradually until they get used to it, and still others who just stick their toes in and then take time to decide if they really want to go swimming or not. By observing, she was “sticking her toe in”. Pushing her in would not have been a good choice on my part.

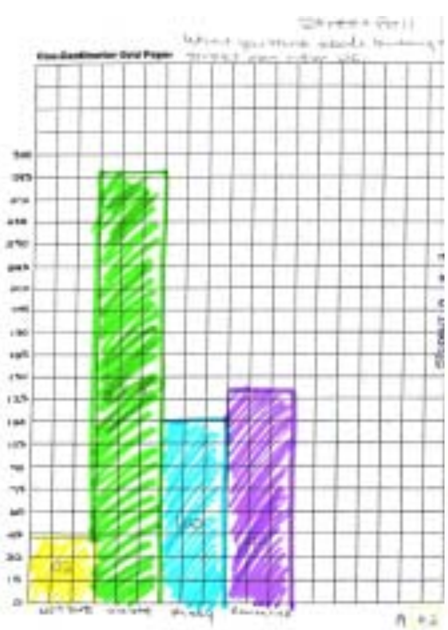
The class asked if they <u>had</u> to do the second graph from the article. There were two circle graphs on the original example so they were certain that there needed to be two bar graphs. I said, “Yes, we can do that but before we do, let me ask you to think about something. Do you need to go all the way up to 600 on the y-axis?” (I had noticed that several of the students had extended their y-axis all the way to 600.)

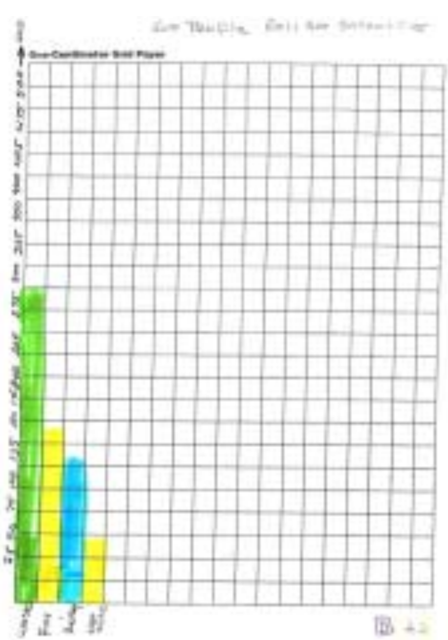
Every student, except for one, agreed that if the 600 wasn't on the graph, no one would know how many people had been polled. Student E (Sharon), who had been observing all this time, was the one who disagreed. When I asked her to explain her thinking she said, "I'm not sure if this is right, but I think you only have to go up as high as your biggest answer, or I mean your biggest number of votes. There aren't any answers that go all the way up to 600." Another student commented, "Then how would you know what the whole is?" Sharon, the quiet observer, then backed down and said, "Yeah, I guess you do need the 600."

Student justifies her reasoning by communicating her answer to her peers.

The students got started on the second bar graph (with more confidence this time). Then I heard one of them say, "Wait a minute, you ***don't*** have to go all the way to 600! You really have to go up to around 300 because that's the highest answer. All the other answers are less than 300. Sharon was right! The way to know how many people were asked is to just add up each of the bars on the graph, they should add up to 600." (For me, the very best part of that observation was "Sharon was right". Peer validation is very powerful.)

The second group of graphs showed more variation, especially on the y-axis. For example, look at Student A's graph below on the left compared to her earlier graph (on page 47). Notice that her y-axis did not go up to 600. Also note Student D's y-axis. Even though it does go up to 600, in this graph she is now using equal increments (see first graph of Student D on page 48.)
As we examined the second set, the students were much more willing to explain their choices. One observation made was that the first graphs looked "smaller" than the second graphs. One student said, "I don't know exactly why one looks bigger than the other but I'm SURE it's got something to do with going all the way up to 600 and not going up to 600." (This will be the topic of our discussion when we return to class next week.)

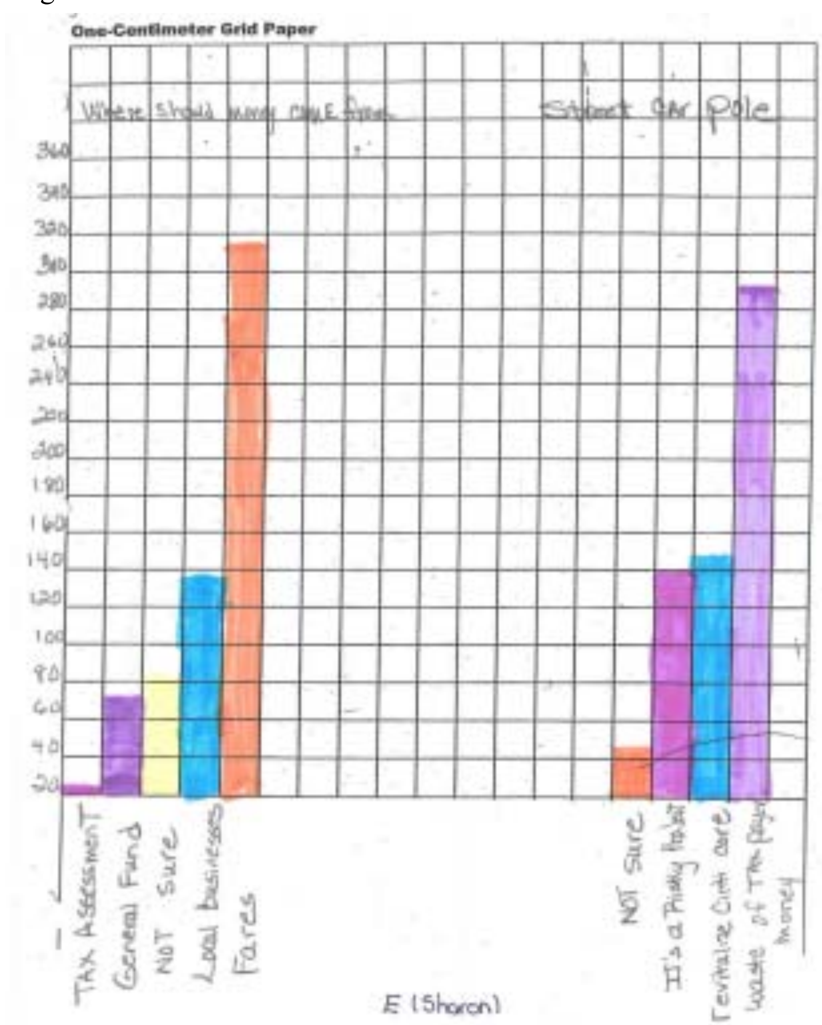




My daily goal is for them to walk out of class feeling as if they learned something and hopefully wanting to learn more. I am aware that some gaps still exist in their understanding, but I think that creating their own models from meaningful data will help them look at other graphs more critically.

Students posed questions to their peer and that student justified her reasoning.

By the way, Sharon (the quiet observer) did complete the task. She worked quietly, quickly, and confidently. She chose to place both bar graphs on one piece of paper, moved in equal increments, labeled and titled everything and was more than pleased with the end result. The other students were impressed and asked her a few interesting questions. One person said, "I didn't think you could put two graphs on one page." Sharon responded by saying, "I thought about that and figured that since the circle graphs were both on one page, there wouldn't be any reason the bar graphs couldn't be, as long as I could make them fit."



Someone else felt that she should have also placed a y-axis on the right hand side of her graph. She explained, "I thought about that too, but if you just trace the numbers on the left all the way over, you can still read the second graph." I think Sharon has learned a lot about bar graphs and I'm very glad I waited her out.

Does It Make Sense? How Do You Know?

by Marty Lopinto

Marty Lopinto teaches at Great Oaks Career Development Campus in Cincinnati, Ohio in the ABLE program. Her students range in age from 18-75 and encompass all 6 literacy/numeracy levels. Most of her students are attending to improve their skills to pass their GED or enter a postsecondary program. Marty's degree in education is from the University of Dayton. She has been teaching adults for the last 16 ½ years. She is also an instructor at Pierre Foods, a local meat packing and processing plant. She runs the Learning Center year round where she helps adults improve their math, reading & writing skills for company advancement, GED attainment, college readiness, and personal satisfaction. Employees come before work, after work or during breaks to work on their individual goals. She says, "I truly love what I do! I feel I am making a big difference in many people's lives!"

In 2005, I participated in the TIAN project for the state of Ohio. One student in particular (Maria) was really struggling with understanding fractions, decimals, percents and solving word problems. Using the TIAN philosophy of discovery and teamwork, I presented activities, including visual representations, for the classroom to explore. Maria then began drawing circle graphs and charts to help her figure out the answer to a problem. Math started to make sense to her. She started to use visuals when figuring out a variety of problems. She starting drawing circle graphs when she encountered percent problems – especially when percent was a benchmark fraction (see sidebar) or close to a benchmark fraction. She went on to pass her GED and enter college.

***Benchmark fractions* are common fractions, such as 1/2, 1/4, and 3/4. These benchmarks can be compared to 'less friendly' fractions such as 11/20 or 4/7.**

From this point on, I was convinced that other students could benefit from this same method of learning and teaching and overcome the dreaded fractions and percents they stress over. Students began to gain confidence, develop strategies to solve problems and have actually come to enjoy math. Here are some examples of activities we do.

Make It Fun, They Will Come

First of all, it is important to have enthusiasm for math. My philosophy is: "If you make it fun, they will come." Because I am so excited about teaching math, the students can feel the energy and want to be a part of the lesson. They enjoy opportunities to come up to the board and explain their answer. At first, some students are uncomfortable with this, but as others come

Productive disposition is a critical element of mathematical proficiency.

Fractions Rap

Fractions, Fractions
Part over Whole
Numerator, Denominator
Fractions are Cool!

Add or Subtract
Find a Common "D"
Multiply, Divide
Cancel Me.

If you divide,
Here's a little tip
First fraction stays the same
The second does a flip.

Fractions, Fractions
Part over Whole
Numerator, Denominator
Fractions are Cool!!!

forward and a student becomes more confident, more and more students are willing to take the chance. If they present something wrong, the other students get involved and discuss why they think the answer is not correct. If needed, I help lead them through to the right answer with questioning that encourages them to take a chance. We always cheer or clap for all of us for discovering the answer.

I had one student, Mike, who really struggled with remembering how to add, subtract, multiply, and divide fractions. Like Mike, many of my other students couldn't remember the rules when using all four operations of fractions. Mike was musically inclined so I made up a rap (see sidebar). As a class, we would rap it. It was fun and was helpful. Students used the rap as a reference sheet. Mike and his classmates said that when they took tests, they would rap that song in their heads so that they could remember the steps.

Working in groups is always fun. I try to pair up stronger math students with weaker students and I walk from group to group asking questions that will help lead them to their own answers. I never tell them how to do it. If they think they have the answer, I again ask, "Does this make sense? How do you know?"

Number of the Day

One great way to start off a math lesson is with Number of the Day. You choose any number and students are asked to write down expressions that are equal to that number. For example, let's say the number is 16. I write it really big on the board and give them one or two minutes to write expressions equal to 16. Then I call on students to share. The first student may say, "8 times 2". I say, "Did you write it like this: 8 X 2? Yes? What is another way?"

Another student says, "With a raised dot, 8 * 2." I continue, "Yes, is there another way?" Someone then offers, "(8)(2)", to which I might say, "Oh, so

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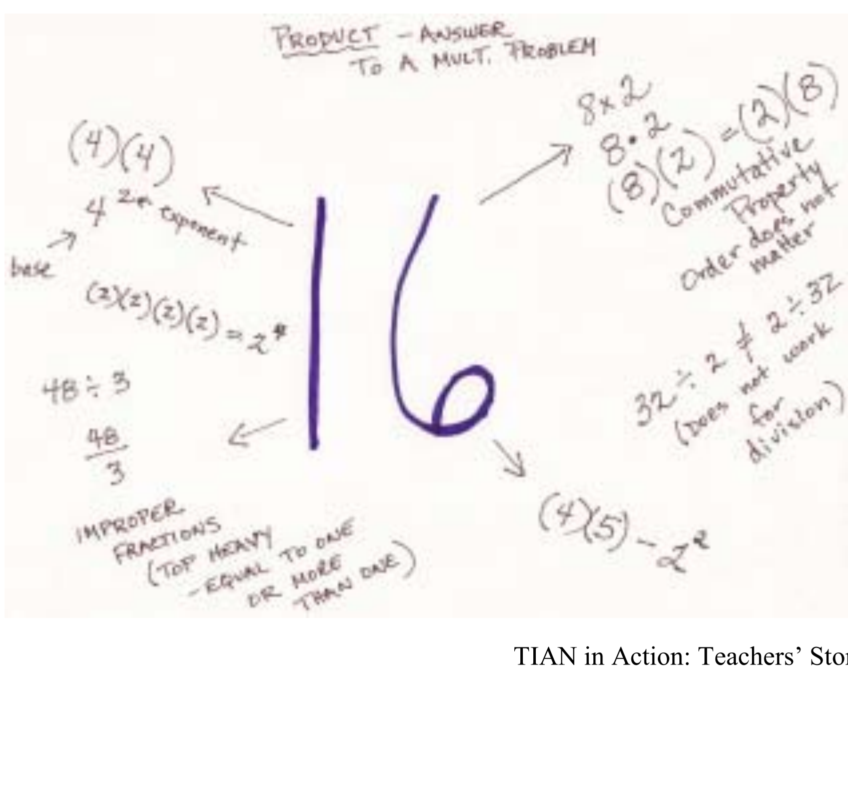


numbers in parenthesis next to one another is multiplication (to reinforce notation conventions)?"

You can even have the students think of a fraction that is equivalent to 16. One student's response was, "48/3". That suggests that he was practicing his multiplication and division at the same time he was creating a fractional representation. This can lead to a conversation about improper fractions, and the idea that the slash symbol (/) means division too.

Depending on the group, I can stay simple or get as complicated as I want. When I first started this activity, students wrote fairly simple expressions to the Number of the Day. As they grew more comfortable, they were writing things like $(4)(5) - 2^2$. Also, a lot of vocabulary can be introduced in this simple activity that only takes five to ten minutes. Most importantly, all students, no matter what level, can actively participate in this activity. This is especially effective in an ABLE classroom where attendance varies daily.

In Number of the Day, the basics of algebra are being incorporated. I find that many of my adult students have never taken an algebra class and are frightened by the word ***algebra***. At the end of this activity, I always say, "Do you know you are doing algebra?" The students then start building confidence with algebra and start convincing themselves that they can be successful and do this. They start learning the rules of math such as the order of operations, square root, exponents, and commutative, associative, and distributive properties. Math starts to become fun and a puzzle to solve!

Benchmark Fractions, Decimal, Percents

If students can visualize decimals, fractions, and percents in their daily lives, they can make sense of them and learn to apply them to everyday situations and story problems they encounter on standardized tests. They make connections to the real world and everything starts to make sense. I always teach them together. From this, students can learn to use a variety of methods to solve a problem.

One hands-on activity that I do so students can visualize what they look like and how they mean the same thing is to hand out a sheet of eight congruent circles and refer to these circles as pizzas. I have one bigger circle they can fold as we go along. Markers and crayons are available for shading. Students become interested because they are ***doing*** and not just learning following rote procedures in a textbook. Students begin to smile.

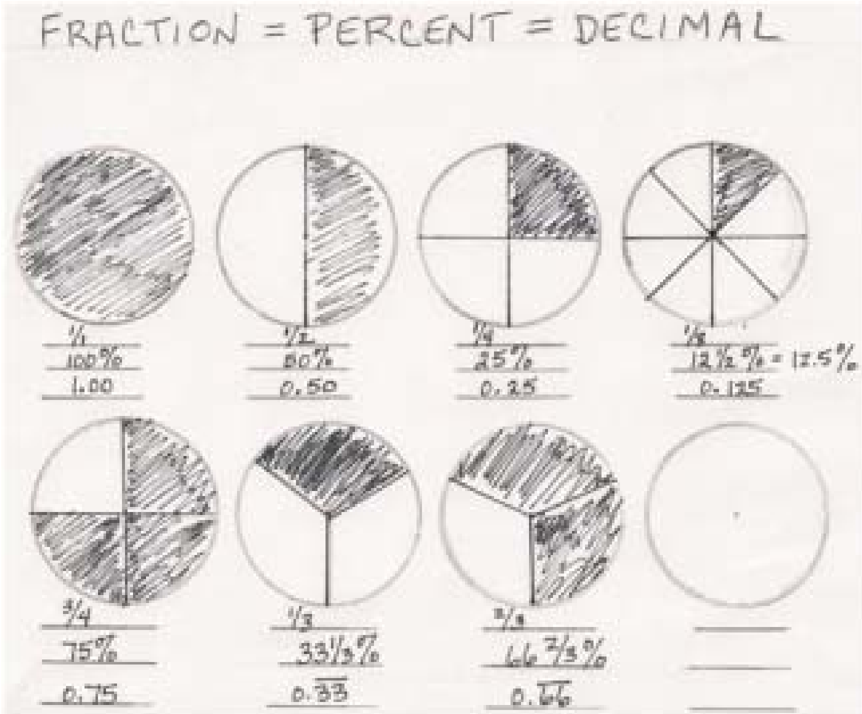
I begin with 100% and its equivalents 1 (or 1/1) and 1.00. After they shade in the entire circle, we move to ½ and its equivalents. All three quantities are always written underneath the circle. I call them a family. They all mean the same.

From ½, we move on to ¼, which is half of a half. Students make connections to ¼ and .25, especially as it relates to money. Then we look at ¾, 75%, and 0.75.

Decomposing numbers helps students develop number sense, which, in turn, helps to build strategic competence.

"Algebrafying arithmetic" is an effective way to begin down the path of algebraic thinking. Students should be informally—but explicitly—exposed to number properties (such as the commutative property, the associative property, the 0 identify for addition) as they develop their number sense.

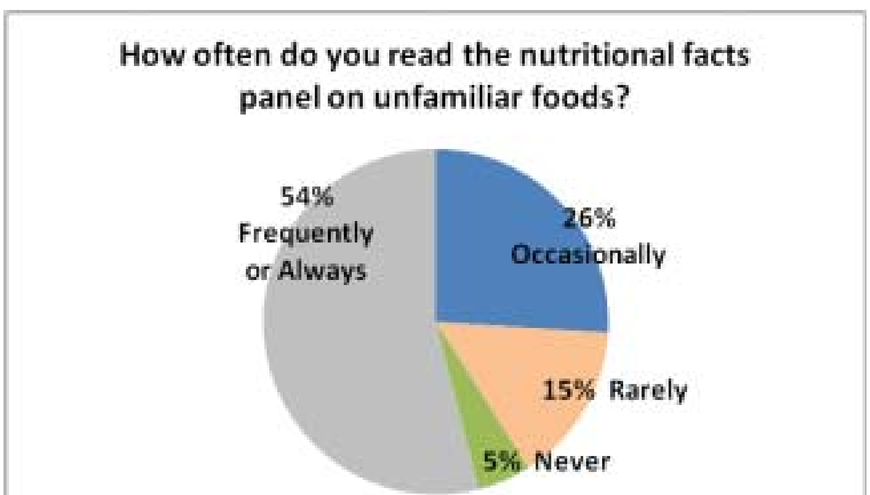
Conceptual understanding includes understanding the relationships among fractions, decimals, and percents so students can fluidly move from one representation to another.



We eventually continue on with an eighth. That is half of a fourth so to follow the pattern, that is ½ of 25% which is 12.5%, and ½ of 0.25 which is 0.125. One-eighth is equivalent to 12.5% and to 0.125.

After students have had plenty of exposure to these benchmarks, I then incorporate those into situations in which students have to find the reasonableness of an answer. I take graphs from a variety of sources (the graph below was adapted from USA Today) and ask questions about them.

Teacher helps students make connections to real-life uses of fractions and percents.



Deloitte Consumer Food Safety Survey of 1102 adults

I ask about each category, “Is it more than ½ or less than ½? How do you know? Does it make sense?” We move from estimating which categories are close to ½, then to ¼, ¾, and even to ⅛.

We then do some estimating using the total surveyed (1,102). Students are able to use their understanding of benchmarks to figure out 50% (or ½) of those were polled, and then make reasonable estimates when given five answer choices (such as on the GED Test). For example, "How many adults said Frequently or Always? A. 550 B. 904 C. 595 D. 492 E. 232"

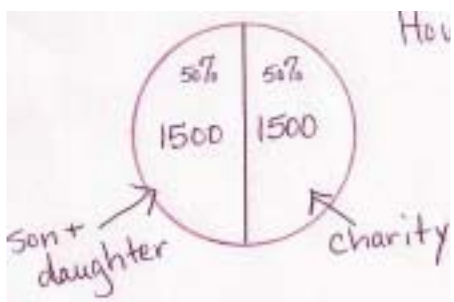
Benchmark percents are very useful for estimating and checking for reasonableness.

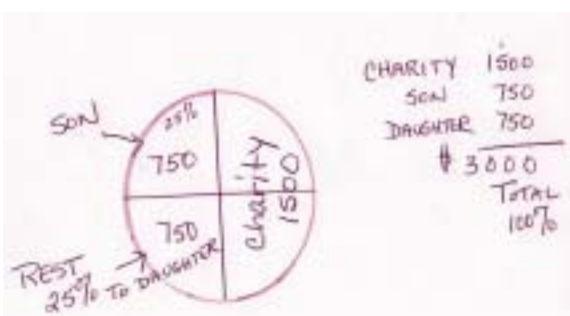
Students also use their understanding of benchmarks to estimate about how many people responded with the response "Occasionally"(26%). By knowing strategies with benchmark fractions, decimals, and percents, students can use the process of elimination when test taking – evaluating the reasonableness of an answer.

Then we start to apply everyday problems using these benchmarks and visualization in situations such as the following example:

> **If Jim has $3000 and he gave 50% to charity, 25% to his son, and the rest to his daughter, how much did he give to each?**

Here is one student's visual explanation of the situation. Notice that she began by first breaking the whole into halves and then split one of the halves into halves again.



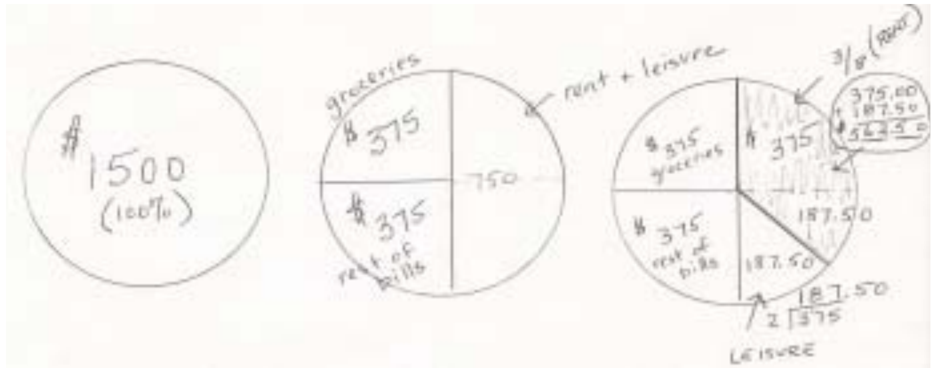


Student's strategic competence is evident in her ability to represent the situation visually and then figure out the solution.

Here's another example:

> **If you make $1500 per month and you spend 3/8 on rent, 1/4 on groceries, and 1/4 on the rest of your bills, how much will you spend on each? How much will you have left for leisure?**

Below you can see how one student is using his benchmarks to go from the whole to a half, to half of a half, and even to eighths.



Students share their answers but also talk about the strategies that they use.

Students are asked to show how they solved the answer and they share their strategies with each other. I repeatedly come back to these benchmark fractions a few times a week so they can get more and more comfortable with them in everyday life.

After students are feeling comfortable with these benchmark fractions, I move onto ⅓ and ⅔. Students are again encouraged to draw these two new benchmark fractions. I then point out three students and say, “If the three of you had to share a dollar equally, how much would each of you get?” One student usually says, “$0.30.” “Does that make sense?” I ask. One student may say, “No, that only adds up to $0.90.” Another student may say, “$0.33 with a penny leftover.” “Is ⅓ more than or less than ¼,? How do you know?” I question. “Just by looking at it on the circles.” Another student may see that 33 ⅓% is more than 25%, or that 33 cents is more than 25 cents.

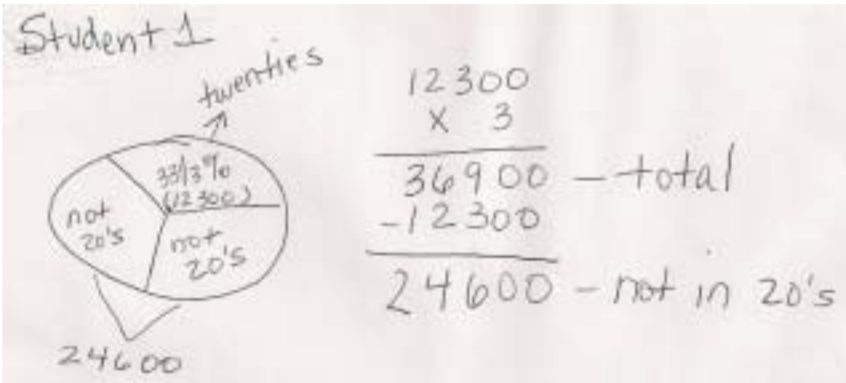
Since students now have a new set of benchmarks (the ⅓ and ⅔ “families”), they again practice applying them in different situations such as the example below.

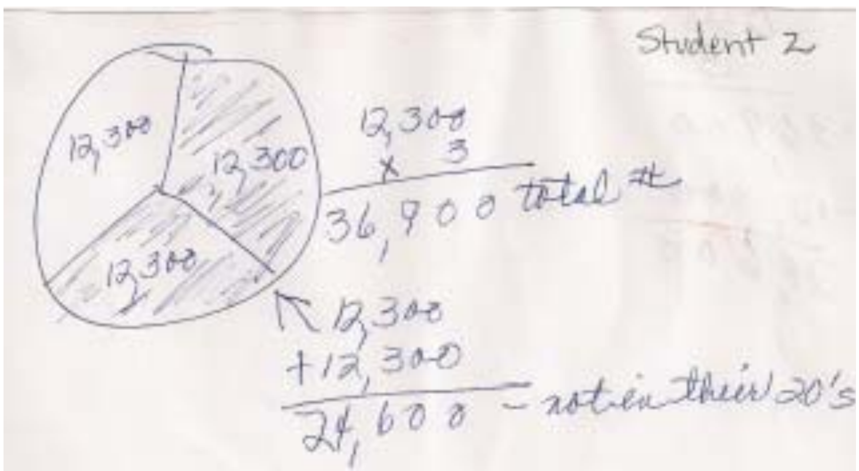
> **James went to a concert at Riverbend. 33 1/3% of the participants were in their twenties. This represented 12,300 people. What total of participants were there at the concert? How many were not in their twenties?**

They are asked to solve it at least two ways and be able to explain their answers to the class. They are encouraged to work with a partner to come up with two strategies.

Student 1 multiplied 3 x 12,300 to get the total, realizing that it took 3 thirds to make the whole amount.

Student 2 used the understanding that 33 % was equivalent to ⅓ so he drew a circle and divided it into three parts. He put 12,300 in each of the three parts and then added them up to get a total. He explained that represented all of the participants. To find out how many were not in their twenties, he took two parts and added them together.





Students develop strategic competence as they learn that different strategies can be used to solve problems. Benchmark percents work well as a strategy in situations in which the numbers are "friendly".

As a class, we then compare the two story problems. I explain that sometimes you are given the whole and asked to find parts (first problem) and other times you are given the part and asked to find the whole (second example). Drawing a circle graph may help determine whether you are looking for the part or the whole.

Drawing graphs is one strategy that students can add to their repertoire.

I do a variety of other activities in my class as well. Sometimes we create informal surveys (such as different hair color of students) so students can practice using benchmarks visually.

On a daily basis I also do a math review where students have opportunities to practice their skills on different problems. Students talk to one another, sharing their strategies. This serves as a confidence booster and also helps to bring new students in to the conversations.

I have noticed that students' TABE scores improve a great deal in a short amount of time. For example, I had one student go from a 5.4 in Applied Math to a 9.2 in three months time. Other students who had not been able to pass the Pre-GED Practice finally developed tools to help them narrow down answers and make sense of math problems; they eventually passed the GED Math Test. Students who are in my class for math remediation for college make great strides as well and learn how to figure out the reasonableness of an answer, gain confidence and test taking strategies, and are able to pass their college entrance tests.

The more students investigate and explain how they came up with an answer, the more they make sense out of a story problem. I see adult learners become more successful and more willing to figure out an answer. The best benefit of all of this is the confidence that they gain and a willingness to try different strategies. All students improve!

One of my students, Mike, summed it up this way:

> *What helped me was having a teacher who struggled with me and actually tried to help. You seemed like you really care about all of us doing something with our lives. I still don't fully understand a lot of math, but* ***what you helped me with was looking at not what I don't know, but what I do know****, and using the process of elimination. I still struggle with math, but at least I can kind of break the problem down in my head.*

Students develop a productive disposition as the teacher acknowledges and builds on what students already know and can do.

Ed. Note: Just before going to press, Marty learned that Mike scored in the 500's on his GED math test. He is planning to start college in the fall. Marty wrote, "We are all psyched. He learned how to critically think through the problem and use what he knows to pick a reasonable answer. He still struggles with basic addition and subtraction but can use 'Does it make sense?' to come up with his answers."

Fostering Communication in a Multicultural Math Class

by Abby Magee

Abby Magee teaches pre-GED Math at Notre Dame Education Center in South Boston. She explained how she found her niche in teaching math: "As I was growing up, my father taught me his love for the beauty and fascination of math. At a young age, I learned to play chess. Today, I still tutor children whose parents want their kids to love math and have fun with it as I did with my father." After graduating from college with a BA in math, Abby went on to get an MBA. She spent 20 years in accounting and finance before realizing that she really wanted to be a math teacher to teach math as she was taught by her father. She went back to school to get an M.Ed. in middle school math and became a 6th/7th grade math teacher before moving into adult education.

I began as a participant in the TIAN Pilot in Massachusetts in 2005, and was struck by the emphasis on learning from one another as we teachers worked on math activities together. I wanted to encourage that same emphasis in my own class but, I was also aware that communication is a challenge for some of my students. My classes are very diverse. They are not only multi-level, but also multicultural, including students from Haiti, Cape Verde, Somalia, Sudan, China, and Iraq, to name a few. My students range in age from 18 to 70 years. Some have physical disabilities, others have learning disabilities. Even though it is challenging, I do believe that communication is a critical component of any adult education class. Let me share some examples.

Opportunities for discussion

In class, I'm always looking for opportunities for discussion. I like to give people the chance to learn from one another. They pair up naturally and often speak in their own language. Sometimes, even if they are not sitting together, they will speak in their language across the room to help a student who needs further explanation. They all support each other, always willing to explain in their own words. It amazes me how even when students come from different language backgrounds, they work so well together, communicating freely and openly.

I sometimes limit the materials that I'm passing out so that students can work on problems together. This motivates them to share ideas with one another. It also gives my English language learners an opportunity to practice using the correct vocabulary for various math tools, such as *protractor*, *ruler*, etc.

Second language issues
Since my students come from many different countries, doing math in English is often a new experience. No wonder it is sometimes an effort explaining math aloud. We spend a lot of time on the terminology and multiple meanings of common words.

Through communication, teacher has an opportunity to ensure that math vocabulary is clearly understood.

One example that immediately comes to mind happened when I was teaching averages. We came upon the term *mean*. Because this word is used in everyday language with very different connotations, I was concerned my students would be confused by the term. I wrote the word on the board and then asked them to give me definitions. They gave me different definitions along with a phrase using each one. I wrote their phrases on the board as follows:

mean - a person who is not nice
mean - when you ask, "What do you mean?"

I asked them if they ever heard the term of someone living "beyond their *means*". Most of the students said they had heard of this before. Then I shared the words and phrases: *mean*while, in the *mean*time, and *mean*ing, just to add a few more instances where the word *mean* shows up in our everyday language.

While many of the students were familiar with those uses for *mean*, only one had ever heard the term used in reference to math. We then spent a good deal of time in class learning about the concept, along with how to find the mean of a set of values.

Seemingly simple terms can be challenging, especially for English language learners. Communication between teacher and students and among students helps to ensure that everyone is "speaking the same language" when working on a problem.

Another example that comes to mind is the simple word *left*. We were reviewing how to choose the correct operation in solving word problems. We talked about phrases such as "how many", "find the total", or "find the sum", all of which might be clues that the problem is an addition situation. For subtraction the problems may ask, "find the difference", "how many more", "how many less", or "how much is left". One of my non-native students was confused with the last example, "how much is left". He only knew left and right as directions, not as a remainder. This is another example of the teaching challenges of teaching math to students whose first language is not English.

But, vocabulary is not the only challenge with students in such a diverse class. Many adult education instructors comment on how they refer to money whenever they want to be sure that their students understand a particular concept. But, what if the monetary system in a student's homeland does not contain quarters, for example? I discovered that I need to be more aware of how I reference our monetary system with my students. Even though our system is based on a decimal system and students can connect that to the metric system, they do not always fractional denominations such as a quarter. (For example, the currency of Somalia includes 500, 100, 50, 20, and 4 shilling notes. Coins come in denominations of 5, 10, and 50 cents, but not 25.)

I was teaching a lesson about improper fractions. I asked the students to draw a picture of 5/4. I verbally used the term *five-fourths* but did not write it on the board. Students were drawing circles.

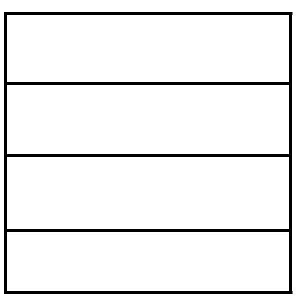
One student took out quarters to draw her circle. She used a quarter so that she could draw nice round circles, not even making the connection that what she was using was one quarter.

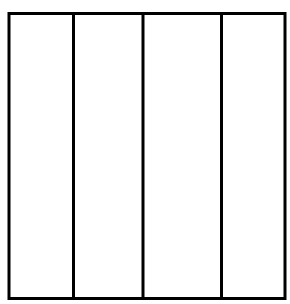
By observing student's visual representation strategy, the teacher gets a view into student's conceptual understanding of 5/4.

Another student drew a circle, divided it into four parts, and then drew one more line dividing one of those parts into two, so that he now had five parts as seen in the picture on the right. I asked him to explain his picture. He said the bottom number was a four, so he knew it first had to be divided into four parts. Then the top number was a five so he needed to add one more part.

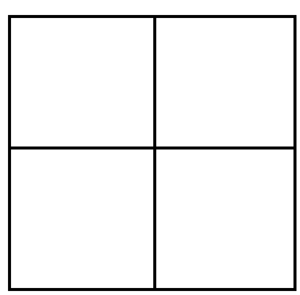
We had quite a discussion trying to use money as the basis for illustrating 5/4, especially since some of the students stated that they did not have a coin representing a "quarter" in their native country's monetary system.

After illustrating five-quarters with our American monetary system, we moved to looking at different ways to divide a shape into quarters. Each student was given a sheet of paper with four 8"x 4" rectangles on it. I asked them to consider the many different ways the rectangle could be divided into four equal parts. Some students just drew lines horizontally and others drew lines vertically on their papers.





I asked if anyone could divide the rectangle a different way, maybe showing four parts that would have equal area, but not necessarily the same shapes. One student came up and first divided the rectangle in half vertically and then split each half horizontally. Students then compared the different "quarters"; whether they are vertical or horizontal slices, or cut both horizontally and vertically. They discovered that they were the same, solidifying the point that four-quarters can be divided in different ways, but all four quarters have to be of equal value.



Student as teacher

One way to get the conversation going is to ask a student to go to the board. I say, “You be the teacher. You explain.”

Initially, they have a hard time with that. They start out shy and uncertain even though they don’t have difficulty working together. Although they come from many countries, they readily work together in pairs or triads. But getting up and explaining math clearly to the whole class stretches them.

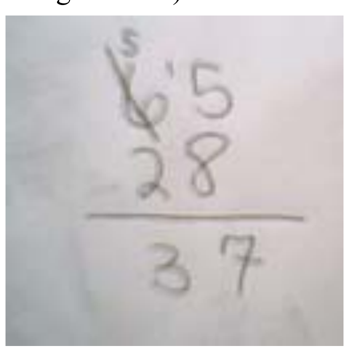
Student errors can be opportunities for rich discussions rather than examples of student failure.

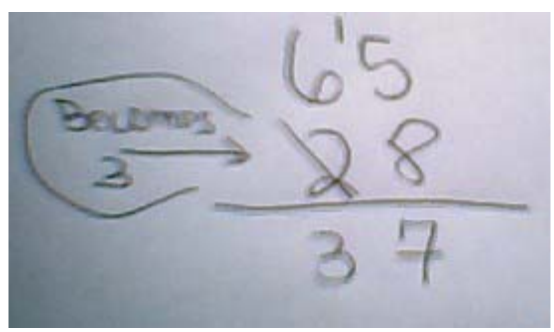
Sometimes they put up an error on the board, like adding both numerators and denominators when adding fractions such as 1/3 + 2/5 = 3/8. If that happens, I let it go for the time being, rather than correcting the error, and inevitably, it turns into a rich discussion. I believe that we learn from our mistakes, and I encourage students not to erase them, but to study them.

Students learn quickly that, although I am the teacher, I am far from perfect. I may make a mathematical error now and then and I am happy for them to point it out. Sometimes students solve problems differently from each other, or from my method. Instead of saying there is only one way to solve it, I make a point of saying that problems can often be solved using different strategies. When I see that students have solved a problem differently, I use this as an opportunity for them to get up and explain their methods. For example, many of my students have learned basic operations such as subtraction and division differently than the way it is typically taught in the United States.

In the U.S., the concept of *borrowing* is used when subtracting a larger amount from a smaller amount. My Haitian, Cape Verdean, and Vietnamese students do not borrow; instead, they add to both the top and bottom numbers. Look at the difference between the way most American students subtract (on the left below) vs. a strategy that does not involve borrowing (on the right below).

Adult learners come with their own strategies. The teacher values these and encourages students to build on what they already know.

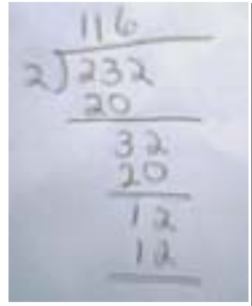




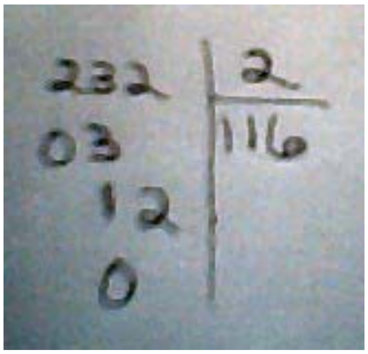
What we might be thinking as we subtract with borrowing is, “We can't take 8 from 5, so we borrow 1 (ten), from 6 (tens). Then we put the 1 in front of the 5 calling it 15. Now we can take 8 from 15, which leaves 7, and 2 from 5 leaves 3.”

My Haitian student might be thinking, “We can't take 8 from 5, so we add ten to both numbers (65 and 28). But rather than make 65 into 75, we make it

60 + 15 so that we can subtract 8 from 15. Since we added ten to both numbers, 28 is actually now 38, so 3 from 6 leaves 3.



Division is also done differently by my Haitian students. Look the U.S. example (on the right) where the divisor is on the left side with the dividend on the right and the quotient on top.



In the Haitian method **(on the left)**, the divisor and dividend are written in reverse positions, and the quotient is written beneath the divisor. Then, like in the U.S., the dividend is divided by the divisor to get the quotient, and then the quotient is multiplied by the divisor, but this is not written down. It is always done in the head and then only the difference is written down, which is to be continuously divided by the divisor until done.

There are other cultural differences as well, such as the reverse use of decimal points and commas. To separate the main monetary unit from the hundredth part, we use the decimal point in the U.S. In many other countries, the comma is used instead. In the U.S., we use commas to mark off whole numbers in groups of three digits for thousands, millions, billions, etc. In other countries, this might be written with a decimal point to separate the three digit groups instead:

Teacher is knowledgeable about differences in how math operations are performed and how amounts are represented.

Whole Numbers

In the U.S. - two million, five hundred thousand (2,500,000)

Other countries (including European countries) - two million, five hundred thousand (2.500.000)

Decimals

Dollars and cents in the U.S. - $3.25

Gourdes and centimes in Haiti - 3,25G

When students act as teachers, I always learn new things from them. It is rewarding for all of us.

Voting

In order to encourage all my students to share, another strategy I use is "voting". After students solve a problem, I accept all their answers, writing them all on the board or on post-it notes. As I am serving as scribe, students start talking about the answers (See MLOTS video at **http://mlots.org/abby/abbypage.html**) and begin to decide on what makes the most sense. After allowing the discussion to proceed, I call for a vote. By that time, a reasonable answer rises to the top. All students have had a say and their responses have been captured, but by putting them randomly on post-it notes,

Voting can be an effective teaching strategy to encourage all students to communicate their solution.

they don't feel that they have to stick with the answers they first offered. Their voices have been heard, but they also are encouraged to consider whether their original answers seem to be most reasonable. I'm fortunate to have a relatively small class where we are close, and I can spend time with each individual student as needed. We have fun coming to consensus, and the "Aha!" moments are thrilling.

I still see and work with other teachers from the TIAN Project. We share our classroom stories and inform each other of new resources available. I have run workshops for teachers such as "Fun with Fractions" and "Algebra for All" where not only am I teaching, but I am seeing new and interesting approaches that these teachers have taken in their own classrooms. There is always more to learn.

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