

Integrating Arithmetic and Algebra

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This article is based on a workshop titled But My Students Don't Know Their Basic Facts, given at the COABE Conference in Philadelphia, PA

Students who have not mastered their basic facts should not be held back from developing deeper critical thinking skills. You can integrate basic arithmetic and algebra, even with students who are working on basic addition facts.

Research into how children learn math suggests that one reason students have difficulty moving from basic arithmetic to algebra is because of their notion of what the equal sign means.

Rather than thinking of the equal sign as a sign of equality – what's on one side of the equal sign is the same as what's on the other side, students think of the equality sign as an indication to perform an operation.

Students are used to seeing computation facts written as $8 + 4 =$ so often, they become attuned to simply reacting to the operation once they get to the equal sign. Because of this, students in many grades (including many of our adult learners) have difficulty with problems such as the one below.

	Answers in Percents			
Grades	7	12	17	both 12 and 17
1 - 2	5	58	13	8
3 - 4	9	49	25	10
5 - 6	2	76	21	2

$$8 + 4 = \square + 5$$

According to Carpenter, Franke, and Levi, students, when asked to solve the above equation, responded as follows: students in grades 5 – 6 gave 12 as an answer (76% of the students) more often than students in grades 1 – 2 (58% of the students).

Strategies to address the equal sign issue

Give students many opportunities to work on basic facts that force them to look at both sides of the equation. Help them recognize that the equal sign indicates equivalence on both sides, not an indication that some operation must precede the equal sign.

One way to encourage students to think about the equal sign is to begin with true/false statements.

$$2 + 5 = 7$$

$$2 + 5 = 5 + 2$$

$$7 = 7$$

$$7 + 0 = 7$$

$$7 = 6 + 1$$

Students need to be able to state whether the equations are true or false and WHY. Being able to articulate the reasoning behind basic facts can lead students to developing their own informal set of “laws”.

For example, $7 + 0 = 7$ is true because “when you add 0 to a number, the number remains the same.” This is a key fundamental principle applied in algebra [additive identity].

Another example: “ $2 + 5 = 5 + 2$ is true because you’re adding the same thing; the numbers are just reversed” [commutative law of addition].

Once students see that these “laws” apply to all the numbers they use, they can develop laws that are more formal:

$$a + 0 = a$$

$$a + b = b + a$$

From true/false statements, move to open-ended statements:

$$2 + \square = 7$$

$$2 + 5 = 5 + \square$$

$$7 = 7$$

$$\square + 0 = 7$$

$$7 = \square + 1$$

$$1 + 6 = 2 + \square$$

In exploring basic facts in this way, students have a better chance of seeing relationships. When basic facts are shown in isolation (flash cards), they do not have the opportunity to see relationships among sets of numbers, for example:

$$2 + 5 =$$

$$5 + 2 =$$

When shown together, students begin to see that $2 + 5$ is just $5 + 2$ reversed. Seeing such patterns over time readily leads them to trust some of the “laws” of algebra.

Eventually, you need to help students transition to more difficult numbers. As they work with “uglier” numbers, they should be encouraged to look for relationships between sets of numbers. Look at the following example:

$$46 + 77 = 44 + \square$$

Students may typically first add the two numbers on the left ($46 + 77$), then subtract 44 from the total to figure out the missing numbers. As students develop their ability to see relationships, they should be able to notice that 44 is 2 less than 46, so the missing number must be 2 more than 77. This type of thinking is critical for students to succeed in more structured, abstract algebra classes.

Try a couple of these yourself, then begin to use all the strategies, one at a time, with your students - no matter at what level they are working.

$$98 + 56 = 99 + \square$$

$$123 + \square = 126 + 87$$

$$416 + 107 = 420 + \square$$

$$\square + 967 = 234 + 964$$

$$369 + 83 = \square + 88$$

Carprenter, T., Franke, M., and Levi, L. (2003). Thinking mathematically: Integrating arithmetic & algebra in elementary school. Heinemann (www.heinemann.com)