Revisiting Subtraction

By Donna Curry

For years, many of us learned that subtraction meant "take away". We learned certain "tricks" to handle those gnarly word problems. We learned to look for words such as "less", "left", "difference", or "decreased by", to decide whether we should subtract or not. And, we learned that when we subtract, we always take the smaller number from the larger number. [Any of us who has overdrawn on our checkbook knows that the larger number doesn't always go on top!] Our adult learners have learned many of the same notions about subtraction. It's important, especially for those students who struggle with "basic math concepts", that they revisit what subtraction is really all about.

In many cases, subtraction can be considered a situation in which we "take away" something. If I had $200 in my checking account and now have $75 left, we could use subtraction to find how much I "took away". But what gets taken away when we compare two quantities? What gets "taken away" if the high temperature was 32 degrees and it is now 22 degrees? What gets "taken away" when we talk about the difference in our ages – I was born in 1985 (I wish!) and you were born in 1979?

Nothing is "taken away" in those examples. When we look at our ages, we're comparing two quantities. So, subtraction also is used to show the difference between them. If students don't realize this, they may not think of such a situation as a subtraction problem. Whether they count up from 1979 to 1985 or whether they subtract 1979 from 1985, they need to understand that comparing two quantities can be solved using a subtraction strategy.

And, what about this kind of situation: I had $42 in my wallet and hurriedly pulled some cash out to give my son at the store. When I got home, I realized I had only $12 left. How much money did I give my son? In this case, we took something away, but we don't know how much, so it's not the same sort of "take away" problem as this: I had $42 and gave my son $30. How much do I have left? The first example of the wallet problem is sometimes called a "missing addend" problem.

While it doesn't matter what we call the different types of problems that can be solved using a subtraction strategy, it is important for students to explore the various situations involving subtraction. In some situations, the beginning amount is unknown, in other cases, that resulting amount is unknown, and in yet other situations, the amount that changed between the beginning amount and result is unknown. That's why it's important that we don't have students jump too quickly to conclusions when looking at key words. Let's look at one "take away" (now sometimes called "separator" type) problem.

> Myrna gave $50 to Jim. She now has $30 left. How much did she have to begin with? [x - 50 = 30] In this example, the beginning amount is unknown.
>
> Myrna had $80 and gave $50 to Jim. How much does she have left? [80 – 50 = x] In this example, the result is unknown.
>
> Myrna had $80 and gave Jim some money. She now has $30. How much did she give Jim? [80 – x = 30] In this example, what's missing is the change between the beginning amount and the result.

All three problems above could be solved using a subtraction strategy. What's interesting to note is that you could also solve each of those problems using an addition strategy since the two are inversely related (and

that's an important BIG idea in math). So, one situation doesn't necessarily require that you subtract vs. add, but probably either will work, depending on the strategy you use. (***Adding up***, or counting on, one number to another can be an efficient subtraction strategy!)

Let's take another look at the problems, focusing on the "key word": ***left.***

Myrna gave $50 to Jim. She now has $30 ***left***. How much did she have to begin with?

Myrna had $80 and gave $50 to Jim. How much does she have ***left***?

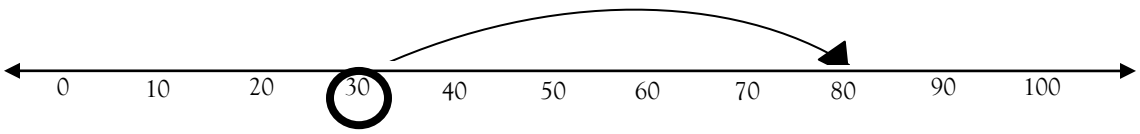
Myrna had $80 and gave Jim some money. She now has $30. How much did she give Jim?

If students only learn to look for "key words", they may try to solve the two problems above using the same exact strategy, in one case subtracting 30 from 50, and the second case, 50 from 80. And they'll be lost with the third problem because there is no "key word".

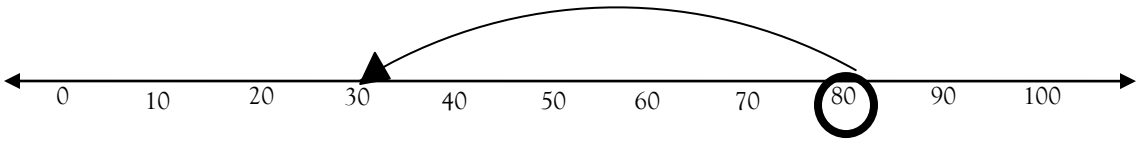
Instead, we need to help students understand each situation and the sort of answer to expect. Ask students to anticipate whether the answer will be more or less than what Myrna has now. Developing an expectation of the solution will help students determine the strategy to use.

One effective strategy that we sometimes ignore when we teach adults is visualization. Visualization can rely on actual manipulatives, such as play money, two-colored chips, etc. Visualization can also take the form of pictures or drawings, including number lines.

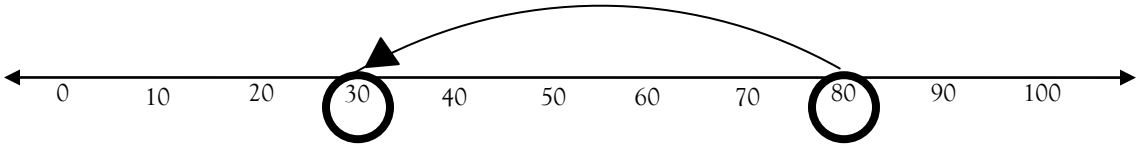
Let's look at Myrna's money problem using a number line. The number line helps students visualize the problem so that they can decide which strategy to use to figure out what's missing.



Myrna had $80 and gave $50 to Jim. How much does she have left?

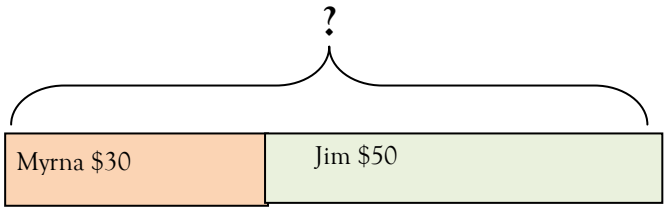


Myrna had $80 and gave Jim some money. She now has $30. How much did she give Jim?

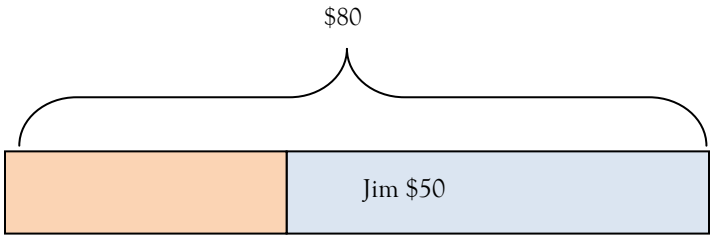


Recently, there has been an increasing interest in how math is taught in Singapore. Beginning very early, students are taught to visualize situations. In doing so, the operation or strategy to use becomes almost obvious. Let's just take a peek at how young children in Singapore might see our same three problems visually (called strip diagrams)—and what you can teach your adult learners! [By the way, these same strip diagrams are used to introduce basic algebra problems. One way to learn more about Singapore math is to begin with an article such as *Solving Algebra and Other Story Problems with Simple Diagrams: a Method Demonstrated in Grade 4—6 Tests Used in Singapore*, from The Mathematics Educator, 2004, Vol. 14, No. 1, pp. 42—46.]

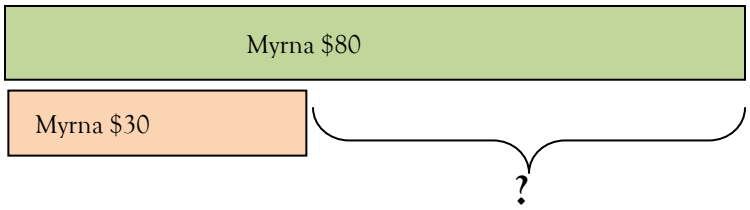
Myrna gave $50 to Jim. She now has $30 left. How much did she have to begin with?



Myrna had $80 and gave $50 to Jim. How much does she have left?



Myrna had $80 and gave Jim some money. She now has $30. How much did she give Jim?



It is important to start with whole numbers with subtraction before moving to fractions or decimals (or – gasp! – signed numbers). Don't assume students really have a clear notion of what subtraction means. Help them visualize the problem first, then they can anticipate what a reasonable answer might look like.

From The Math Practitioner Vol. 15, No. 2